## PLANNING THE LESSON: DESIGNING INSTRUCTION FOR DISCIPLINARY THINKING AND LEARNING

### FOCUS QUESTION
Foster Metacognition

How and when do I apply strategies involving fractions referring to the same whole to solve word problems?

### TASK/PROBLEM
Susan is excited about art class today. Each student is given 4 yards of ribbon to work with. Susan doesn’t want to be wasteful, so she decides to create different art projects using all 4 yards of ribbon that was given to her. Which art projects can Susan make that will use exactly 4 yards of ribbon? Show your thinking in more than one way using models, numbers, and words.

Be ready to explain your thinking to a partner using connecting words/phrases (initially, next, for example, as a result of, specifically, etc.) and math vocabulary.

<table>
<thead>
<tr>
<th>Project</th>
<th>Length of Ribbon (in yards)</th>
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<tbody>
<tr>
<td>Flower</td>
<td>1 1/2</td>
</tr>
<tr>
<td>Bulletin board</td>
<td>3 1/2</td>
</tr>
<tr>
<td>Costume</td>
<td>2</td>
</tr>
<tr>
<td>Mask</td>
<td>1 1/8</td>
</tr>
<tr>
<td>Puppet</td>
<td>2 1/2</td>
</tr>
<tr>
<td>Picture frame</td>
<td>1/4</td>
</tr>
</tbody>
</table>

### LANGUAGE DEMANDS
Monitor and Guide Disciplinary Learning

English Learners will need support with the following:

- **Making sense of the problem (MP1)**
  - Interpreting challenging language – to work with, to be wasteful, to create different art projects, use exactly 4 yards
  - Understanding math vocabulary – yard(s), length, equivalent, equal to, represent, model
  - Identifying the question - “Which art projects can Susan make that will use exactly 4 yards of ribbon? Explaining and justifying their thinking clearly and precisely (MP3 & MP6)
  - See Language Objective, p. 3 and Supports & Structures (Model Constructive Conversation), p. 7-8

### MATH CONTENT STANDARD(S)
Set Disciplinary Learning Targets

5.NF.2: Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $2/5 + 1/2 = 3/7$, by observing that $3/7 < 1/2$. 

## Planning the Lesson: Designing Instruction for Disciplinary Thinking and Learning

### Math Practice Standard(s)

<table>
<thead>
<tr>
<th>Standard(s)</th>
<th>Disciplinary Learning Targets</th>
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<tbody>
<tr>
<td>MP1: Make sense of problems and persevere in solving them (FOCUS MP)</td>
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<tr>
<td>MP2: Reason abstractly and quantitatively</td>
<td></td>
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<tr>
<td>MP3: Construct viable arguments and critique the reasoning of others (FOCUS MP)</td>
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<tr>
<td>MP4: Model with mathematics</td>
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<tr>
<td>MP5: Use appropriate tools strategically</td>
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<td>MP6: Attend to precision</td>
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<td>MP7: Look for and make use of structure</td>
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<td>MP8: Look for and express regularity in repeated reasoning</td>
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### CA ELD Standard(s)

<table>
<thead>
<tr>
<th>Standard(s)</th>
<th>Disciplinary Learning Targets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exchanging Information and Ideas - ELD.PI.5.1. Ex: Contribute to class, group, and partner discussions, including sustained dialogue, by following turn-taking rules, asking relevant questions, affirming others, and adding relevant information.</td>
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</tr>
<tr>
<td>Listening Actively - ELD.PI.5.5 Ex: Demonstrate active listening of read-alouds and oral presentations by asking and answering detailed questions with occasional prompting and moderate support.</td>
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<tr>
<td>Reading/Viewing Closely - ELD.PI.5.6a. Ex: Explain ideas, phenomena, processes, and text relationships based on close reading of a variety of grade-level texts and viewing of multimedia with moderate support.</td>
<td></td>
</tr>
<tr>
<td>Understanding Cohesion - ELD.PII.5.2b. Ex: Apply growing understanding of how ideas, events, or reasons are linked throughout a text using a variety of connecting words or phrases to comprehending texts and writing texts with increasing cohesion.</td>
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### Math Content Objective

<table>
<thead>
<tr>
<th>Objective</th>
<th>Disciplinary Learning Targets</th>
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</thead>
<tbody>
<tr>
<td>Students will be able to apply and justify strategies involving addition and subtraction of fractions referring to the same whole, record results, and justify their conclusions with a model.</td>
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</table>
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<table>
<thead>
<tr>
<th>LANGUAGE OBJECTIVE(S)</th>
<th>Students will be able to explain and justify their solution using connecting words and phrases (initially, next, for example, as a result of, specifically, etc.) and math vocabulary (yards, length, equivalent, equal to, represent, fraction, add, subtract, equivalent, etc.) in a conversation with a partner.</th>
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</table>

POSSIBLE SOLUTIONS
Monitor and Guide Disciplinary Learning

| NOTE: This problem has two possible combinations that equal 4 yards. Susan could make a flower, a picture frame, and a costume as one combination. She could also make a bulletin board and mask. Both solutions are correct. The latter requires students to add fractions with different denominators. |

<table>
<thead>
<tr>
<th>Addition with Fraction Strips</th>
<th>Addition with Friendly Fractions</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="ribbon_diagram.png" alt="" /></td>
<td><img src="friendly_fractions.png" alt="" /></td>
</tr>
</tbody>
</table>

Susan can make a flower, a picture frame, and a costume for her art projects that total 4 yards of ribbon.

- Flower + Picture Frame
  - $1 \frac{3}{4} + \frac{1}{4} = 1 \frac{4}{4}$ (2 yards)

- Flower/Picture Frame + Costume
  - $2 \text{ yds.} + 2 \text{ yds.} = 4 \text{ yds.}$
PLANNING THE LESSON: DESIGNING INSTRUCTION FOR DISCIPLINARY THINKING AND LEARNING

POSSIBLE SOLUTIONS
Monitor and Guide Disciplinary Learning

Addition with Fraction Strips

Susan can make a flower, a picture frame, and a costume for her art projects.

1 ⅓ + ⅓ + 2 = 4 yds.

Decomposing Fractions with Addition

Adding Fractions with Number Line

Subtraction with Fractions Strips
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Decomposing Fractions with Subtraction

Using visuals and creating equivalent fractions with equations

POSSIBLE MISCONCEPTIONS
Monitor and Guide Disciplinary Learning

Students might mix models when adding, subtracting or comparing fractions. (i.e., Students will use a circle for thirds and a rectangle for fourths.

Students may create models of fractions that are difficult to compare because the size of the whole is not the same for all representations.

Students will make mistakes in creating equations when adding or subtracting fractions, i.e. add the numerators and the denominators.

Susan’s Art Project

3 1/3 yards + 2/3 yards = 4 yards

2/3 yards x 2/3 yards = 4/9 yards

Since 4/9 is equivalent to 7/3 and 7/3 yards is all that is needed to make 4 yards, Susan can also make a Bulletin Board and a Mask with 4 yards of ribbon.
<table>
<thead>
<tr>
<th>STUDENT STRATEGIES</th>
<th>FOCUSING QUESTIONS</th>
<th>ASSESSING QUESTIONS</th>
<th>ADVANCE QUESTIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student page is blank with little or no work begun</td>
<td>What do you need to find out?</td>
<td>Why did you decide to use this strategy? Does your strategy make sense?  Why or why not?</td>
<td>What other ways can you solve the problem? Are there any other solutions to the problem?</td>
</tr>
<tr>
<td>[1 \frac{3}{4} + \frac{1}{4} + 2 = 4]</td>
<td>How could you model this problem? What tools could you use?</td>
<td>What do the wholes and parts represent? How do you know your answer is accurate?</td>
<td>How could you express this with an equation?</td>
</tr>
<tr>
<td>Student has the first combination correct. But is struggling to discover that the Mask and the Bulletin Board are also correct solutions.</td>
<td>What could you do to figure out if any other fractions can be combined to make 4 yards of ribbon?</td>
<td>How does the way you created your model help you be more accurate?</td>
<td></td>
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</tbody>
</table>
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<table>
<thead>
<tr>
<th>DISCIPLINARY DISCUSSION FOCUS</th>
<th>Targeted Constructive Conversation Skill(s)</th>
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<tbody>
<tr>
<td></td>
<td>□ CREATE  ✗ CLARIFY  ✗ FORTIFY  □ NEGOTIATE</td>
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MODEL CONSTRUCTIVE CONVERSATION

<table>
<thead>
<tr>
<th>Prompt Starters:</th>
<th>Response Starters:</th>
</tr>
</thead>
<tbody>
<tr>
<td>How did you approach the problem?</td>
<td>To solve the problem, I initially...</td>
</tr>
<tr>
<td>Can you elaborate on that idea?</td>
<td>For my next step...therefore...</td>
</tr>
<tr>
<td>Why did you...?</td>
<td>Afterward... because...</td>
</tr>
<tr>
<td>How do you know your thinking makes sense?</td>
<td>Next.... Finally, ...</td>
</tr>
<tr>
<td>How do you know your thinking addresses the problem?</td>
<td>For example, ... Is that clear?</td>
</tr>
<tr>
<td>What made you think that...?</td>
<td>Specifically, I used... to represent... Does that make sense?</td>
</tr>
</tbody>
</table>

**Visual of Solution:**

![Diagram of ribbon usage](image)

**Rationale:**

The purpose of the Model Constructive Conversation is to provide students with an explicit model of what their own conversation should sound like. The conversation should exemplify how to apply academic language to address the prompt. Models may be crafted to surface misconceptions as well as correct solution pathways.

In this lesson the visual highlights a model that students might create. The Model Constructive Conversation focuses on the skills of CLARIFYING and FORTIFYING.
### PLANNING THE LESSON: DESIGNING INSTRUCTION FOR DISCIPLINARY THINKING AND LEARNING

**Conversation Prompt:** Use your Constructive Conversation Skills to interview your partner about their approach for solving the problem. Focus on **Clarifying** and **Fortifying** each other’s ideas.

**A:** How did you approach the problem?

**B:** My understanding is that Susan wants to make as many different art projects as possible with exactly four yards of ribbon. **Therefore,** I used a tape diagram to represent the total amount of ribbon she has. **Next,** I drew another bar model underneath and decomposed it into each individual yard. **Finally,** I started to subtract art projects from that total amount. **Does that make sense?**

**A:** **What I understood was** that you used fraction strips to build a tape diagram that represents the four yards of ribbon as whole and another that represents the yards for the items Susan might decide to make. Then you subtracted. **Can you elaborate on that idea?**

**B:** **My intention was to** illustrate how the three projects combine to equal 4 yards. **For example,** the two orange fraction strips represent the two yards needed to make the costume, **while** the green strips represent the one and three-fourths of a yard required to make the flower. **Is that clear?**

**A:** **Following the same logic,** the yellow fraction strip represents the one-fourth of a yard needed for the picture frame. Aside from using a tape diagram and a number line, **I noticed that** you also used equations to show your thinking. **Can you tell me what each number represents?**

**B:** **Of course.** The numbers in the first equation represent the two yards for the costume project that I subtracted from the four yards of ribbon. **As you can see,** the difference was two yards. The next equation, I subtracted one and three-quarter yards for the flower project from the left over two yards of ribbon, which left me with one-fourth of a yard. The only art project left that uses one-fourth of a yard is the picture frame, so I ended up using all four yards of ribbon. **What other questions might you have about my thinking?**

**A:** **I heard you say** that your equations show how you subtracted the different art projects from the four yards of ribbon and I would like to add that your equations help me visualize what you did with your fraction bars, as you subtracted. **How do you know your thinking addresses the problem?**

**B:** **I know that** Susan wants to make as many different art projects as possible. **Also know that** she is using exactly four yards of ribbon. **What I needed to** find out is how many different art projects are possible Susan can make with exactly four yards of ribbon. **So I figured** I’ll just subtract the different art projects that used up exactly four yards and came up with three different art projects. **How does my idea compare to yours?**
1. Introduce focus question and objectives of the lesson
2. Review Norms of Interaction and Constructive Conversation Skills

**Say:** Today’s math lesson will help us add to our understanding around our focus question.

**Focus Question:**
How and when do I apply strategies involving fractions referring to the same whole to solve word problems?

**Say:** At the end of the lesson, we will come back to this question to see if we learned any new ideas that help us understand how fractions and decimals are related.

**Refer to Focus Math Practices – MP1 and MP3**

**Say:** Today we will work as mathematicians as we solve the problem. Let’s review our Math Practice goals:

- **MP1** – I can make sense of the problem
- **MP3** – I can explain my thinking and listen/ask questions to understand others

**Say:** We are going to be doing a lot of talking today. During our conversation let’s make sure we use our Conversation Norms and our Constructive Conversation Skills (point to posters). Since we are going to explain our math thinking, we are going focus on the skills of Clarifying and Fortifying. You may use the prompt and response starters to help you if you need them.

**Say:** Let’s review our language objective.

**Language Objective:**
Today I will...
- explain my thinking to a partner
- use connecting words/phrases
- Use math vocabulary
**BEFORE PHASE**

1. Activate prior knowledge
2. Pose the problem
3. Read to clarify language from the problem
4. Ensure that students understand the task and have a plan to begin solving

**Scaffolds:**
- Notice & Wonderings
- Three Reads
- Think Aloud
- Constructive Conversation Skills
- Prompt & Response Starters

**ACTIVATE PRIOR KNOWLEDGE WITH NOTICE AND WONDERINGS**

**Say:** What do you notice? What do you wonder?

Have students share out. Make sure to surface the following:
- Students might say “roll of yarn” but yarn is used for knitting. Make sure students know this is ribbon.
- Students need to know that these are rolls of ribbon and they are used for decorating things or for art projects. It might be helpful to show students what a yard of ribbon looks like.

Have students discuss with a partner. Select one or two volunteers to share their ideas with the class.

**Say:** How might this connect to math?

**POSE THE PROBLEM**

Present the problem to students. Either project it, have it charted, or typed out on paper so that every student is able to see the problem.

**THREE READS PROTOCOL**

**Say:** We will use our Three Reads Protocol to take time to make sense of the problem and persevere to solve it as Mathematicians do. Why would we want to read the problem several times? How will this help us? (MP1) (Have one or two students share out)

**FIRST READ – READ TO UNDERSTAND THE STORY (CLARIFY CONTEXTUAL LANGUAGE)**

**Say:** For our first read we will focus on understanding the story. Listen as I read it to you and try to visualize what’s happening in the problem.

**Say:** Now that we’ve read the problem, have a Constructive Conversation with your partner to discuss the following questions: What is happening in the problem? What are we trying to find out? How do you know?

Listen to students’ conversations. Then, have a one or two students share out with the class. Use questioning to clarify any unfamiliar language and ensure students understand the following:
- Susan wants to make as many different art projects as possible—What does make as many different…as possible mean?
- She will be using exactly 4 yards of ribbon—What does exactly mean?
- What does 4 yards of ribbon look like?
- What are the different art projects listed?
SECOND READ – READ TO UNDERSTAND THE MATH (CLARIFY CONTENT LANGUAGE)

Say: For our second read our focus is to understand the math. You will echo read each sentence after I read it. Visualize the quantities and how they are related.

Say: Now that we’ve read the problem a second time, have a Constructive Conversation with your partner to discuss the following questions: What does each number in the problem represent? How will these numbers help us solve the problem? (MP1, MP2)

Listen to students’ conversations. Then, have a one or two students share out with the class. Use questioning to clarify any unfamiliar language and ensure students understand the following:
- Susan will use exactly 4 yards of ribbon for the different projects
- Understanding the different lengths of ribbon in terms of fractions (thirds, fourths, halves)

THIRD READ – READ TO MAKE A PLAN (FOSTER METACOGNITION)

Say: For our third read our focus is to begin thinking of a plan to solve this problem. We will read chorally in one voice. As we read, think about all the important information that will help you solve the problem. Visualize possible ways to begin solving the problem.

Say: Now that we’ve read the problem a third time, I want you to use your think time to begin planning your approach to this problem.

Give students think time, then do a “Think Aloud” to model how to think of a plan to solve.

Say: I’m thinking of similar problems that we’ve solved in the past that might help me. I remember that we learned how to add, subtract, and compare fractions. We also learned how to find equivalent fractions. I wonder if using what I learned might help me solve this problem. I know I have to model the different lengths of the art projects so I can see which combinations of them are exactly equal to the four yards of ribbon Susan has. Hmmm…what tool should I use? Perhaps a number line, or maybe I can use fraction strips, or an area model. What tool would be the most helpful for this situation? I think I will begin by…(MP1, MP5)

Say: Mathematicians take their time to make sense of the problem and then make a plan to approach the problem, just as we did right now. This is especially important to do when a problem is challenging. Now we are ready to begin solving the problem.
**DURING PHASE**

1. Let go! Allow for productive struggle time
2. Circulate as students work independently first
3. Ask questions to focus, assess, and advance student thinking
4. Circulate as students work in pairs or in groups
5. Collect a language sample
6. Decide which solutions will be selected for sharing.

**Scaffolds:**
- Math Interview
- Model Constructive Conversation
- Constructive Conversation Skills
- Prompt & Response Starters
- Fishbowl

**STUDENTS SOLVE THE PROBLEM**

Hand out materials (paper, manipulatives, etc.) and provide students with independent struggle time to solve the problem and represent their solutions.

**Say:** Now that we’ve made sense of the problem and thought of a plan to solve it, each of you will work on solving the problem independently. Remember to show your thinking using numbers, pictures, symbols, and words. *(MP1, MP2, MP4)*

**TEACHER CIRCULATES AS STUDENTS WORK INDEPENDENTLY**

Circulate and provide individual students with support as needed; prioritize students who need help with an entry point into the problem. A good starting point with any student is to say, “Tell me about what you did here” as you point to their work.

Please refer to the “Planning the Lesson” section of this lesson plan for examples of questions to FOCUS, ASSESS, OR ADVANCE student thinking.

**TEACHER DISPLAYS VISUAL OF SOLUTION AS STUDENTS LISTEN TO THE MODEL CONSTRUCTIVE CONVERSATION**

Introduce the Model Constructive Conversation. See p. 8

**Say:** Let’s come back together. Some of you may not be finished; that’s fine. What is most important, is that you are making sense of the problem and have begun trying to solve it. Now, we will share our thinking with a partner to learn about different ways to solve this problem. Let’s review what we need to do as we discuss our thinking.

Review the LANGUAGE OBJECTIVE with students and present the model.

**Say:** Let’s review the language objective (point to charted language objective as students read it). I want you to all listen carefully to this conversation and listen for the parts where the students Clarify or Fortify their thinking. Use your hand signals when you hear language for Clarifying or Fortifying

Use one of the following options to present the Model Constructive Conversation:
- The teacher and a student each read a part
- A student and another student each read a part
- The teacher uses puppets to read each part
- The teacher and another adult each read a part
- Pre-recorded audio of a male and female each reading a part

Repeat portions of the Model as needed to highlight CLARIFYING and FORTIFYING Language.
• **TEACHER DEBRIEFS THE MODEL CONSTRUCTIVE CONVERSATION**

  **Say:** Let’s think about the conversation we just heard.

  Pose the following questions pausing to have one or two students share out for each.

  ✓ What specific language did we use to explain our thinking?
  ✓ What specific language did we use to make our ideas clearer?
  ✓ What specific language did we use to support our ideas with evidence?

• **MATH INTERVIEW (MP3, MP6)**

  **ROUND 1 – ONE STUDENT INTERVIEWS THE OTHER, THEN STUDENTS SWITCH ROLES (COLLECT A LANGUAGE SAMPLE)**

  **Say:** Now, it’s time to begin our “Math Interview”. Remember some of you will interview your partner first and some of you will be explaining your thinking and answering questions first. Then you will switch roles and go through the process again.

  **Say:** Don’t forget to focus on clarifying and fortifying ideas during your conversation. I will also be listening to your conversations to see who is (refer to language objective) using connecting words/phrases and math vocabulary. Remember to use your Prompt and Response Starters to assist you if you need to use them during your interview. Take some time to review them with your partner.

  Call on one or two students to share one prompt starter they might use and which response starter would be useful for a reply.

  **Say:** I will come around and listen to some of your conversations. I might also be asking you and your partner some questions to understand your thinking. You may begin.

  Circulate and select one pair of students to COLLECT A LANGUAGE SAMPLE. Bring class back together after most students have interviewed each other.

• **FISHBOWL OF STUDENT INTERVIEW**

  Invite a pair of students to come demonstrate how they Interviewed each other.

  **Say:** I heard _____ and _____ doing their best to Clarify and Fortify their ideas during their math interview. Let’s listen to their conversation and try to learn from their exchange.

  Have a student pair demonstrate a few exchanges as the rest of the class listens. Provide positive feedback that may include the following:

  ✓ Making ideas clearer
  ✓ Supporting ideas with evidence (referring to model/representation)
  ✓ Use of academic language
• ROUND 2 – STUDENTS INTERVIEW ANOTHER PARTNER, THEN SWITCH ROLES (DECIDE WHICH SOLUTIONS TO SHARE IN THE AFTER PHASE)

Say: Now, it’s time to begin our second round of “Math Interview”. Remember some of you will interview your partner first and some of you will be explaining your thinking and answering questions first. Then you will switch roles and go through the process again.

Say: I will come around and listen to some of your conversations. I might also be asking you and your partner some questions to understand your thinking. You may begin.

As you circulate, consider which solutions (two or three) you will select for your targeted whole-class discussion (MATH SUMMIT) in the After Phase. Make sure to select solutions based on the objectives of the lesson and the students’ instructional needs.

• STUDENTS TAKE TIME TO REFLECT

Say: As mathematicians we know how important it is to explain our thinking and try to understand the thinking of others (MP3). This helps us really learn and understand important math ideas. I want you to take some time to reflect after going through the math interview process. Use your think time to consider the following questions: What did you learn? What new questions might you have?

You may have students:
- Share their reflection with a partner
- Write in their math journal
- Write on a post it
**AFTER PHASE**

1. Order selected solutions strategically
2. Facilitate the sharing of two or more solution paths
3. Ask questions to facilitate a student-centered discussion
4. Identify patterns and make mathematical generalizations
5. Formalize the main ideas
6. Identify next steps and future problems.

**Scaffolds:**
- Math Summit
- Constructive Conversation Skills
- Prompt & Response Starters

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**MATH SUMMIT (MP1, MP2, MP3, MP4, MP6)**

**Say:** Mathematicians, let’s begin our Math Summit. Who can remind us what we do during our Math Summit and why? (Have one or two students share out.)

**Say:** That’s right. Math Summit is our opportunity to focus on one or two solutions and try to understand the math together. Do your best to use academic language throughout our discussion and to use your prompt and response starters if you need them.

- **STUDENTS INTERPRET THE FIRST SOLUTION**
  
  Present the first solution for students to interpret quietly to themselves.
  
  **Say:** Here is one solution. Use your think time. What do you notice about the solution? Turn and talk.

  Invite the student to come up to explain her/his solution to the class.
  
  **Say:** This is actually _____’s solution. Please come up to explain your thinking. As the audience, the rest of us will listen carefully and try to understand your classmate’s explanation. Be ready to ask questions and discuss. How did you approach the problem? What is the first step you took?

  Use guiding questions to provide the student support as s/he explains her/his solution to the class.

  **Say:** So, how did _____ solve the problem? What was her/his approach? Turn and talk to your partner. (Have one or two students share out)

  Have one or two students ask questions of the presenting student.

  **Say:** Does anyone have any questions for _____?

- **STUDENTS INTERPRET THE SECOND AND/OR THIRD SOLUTION**
  
  Repeat the process with a second and/or third solution:
  - Students interpret the solution
  - Students discuss what they notice about the solution
  - Student comes up to explain her/his approach while teacher provides guidance
  - Students discuss what they understood about her/his explanation
  - A few students share out their understanding of the explanation
  - A few students ask questions of the presenting student

- **STUDENTS COMPARE AND CONNECT SOLUTIONS**
  
  Facilitate a discussion where students compare and connect solutions shared. Make sure to identify similarities and differences across the solutions to highlight key mathematical ideas for the lesson.

  **Say:** Mathematicians, how are these two solutions similar or different? Turn and talk to your partner. Have a few students share out; accept multiple responses.
**STUDENTS REVISE OR ADD TO THEIR SOLUTIONS AND SUMMARIZE THEIR LEARNING**

*Say:* Take a few minutes to consider what you learned from the other mathematicians in the room today, and either revise or add to your original solution using a pen. Don’t erase your original thinking. (Circulate and support students as needed while they revise or add to their solutions.)

*Say:* Let's summarize what we have learned from our lesson today. We saw two different strategies for solving this problem. Which solution path did you prefer? Why? Talk to your partner.

Have one or two students share out and make sure to highlight one or two of the following key mathematical ideas:

- You can compare different art projects to determine which ones might add up to 4 yards by reasoning about their size using benchmark fractions
- You can justify the conclusions by using a visual model such as fraction strips, area model, number line, etc. and also show your thinking in an equation
- You can create equivalent fractions when adding fractions with unlike denominators
- Problems won’t always just have one right answer. Multiple solutions might be possible.

**WRAP-UP & NEXT STEPS**

1. Review focus question and lesson objectives
2. Allow for students to self-assess and monitor progress toward lesson objectives
3. Give feedback to students on objectives that will move their learning forward
4. Close lesson and introduce topic for next lesson

**REVIEW FOCUS QUESTION AND LESSON OBJECTIVES** (should be charted/posted on the board)

*Say:* Let’s go back to our focus question. How did our learning today add to our understanding of fractions? Let’s add these ideas to our chart.

Allow for students to self-assess and monitor progress toward lesson objectives

*Say:* As we review our lesson objectives out-loud, give me a thumbs-up signal if you feel you did this today during our math lesson.

Read each objective out-loud and watch for student self-assessment. Then give feedback to students so students know what they did well and what areas need improvement.

*Say:* I noticed many of you were… Now I want you to think of one thing you will try to improve on for next time. Who would like to share?

Close the lesson and introduce the topic for the next lesson.

*Say:* Based on our learning today, our next steps will be to work on...
**TASK/PROBLEM:**
Susan is excited about art class today. She wants to make as many different art projects as possible using exactly 4 yards of ribbon. Which art projects will Susan make that will use exactly 4 yards of ribbon? Show your thinking in more than one way using models, numbers, and words.

Be ready to explain your thinking to a partner using connecting words/phrases (initially, next, for example, as a result of, specifically, etc.) and math vocabulary.

<table>
<thead>
<tr>
<th>Project</th>
<th>Length of Ribbon (in yards)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flower</td>
<td>$1 \frac{3}{4}$</td>
</tr>
<tr>
<td>Bulletin board</td>
<td>$3 \frac{1}{3}$</td>
</tr>
<tr>
<td>Costume</td>
<td>$2$</td>
</tr>
<tr>
<td>Mask</td>
<td>$\frac{4}{6}$</td>
</tr>
<tr>
<td>Puppet</td>
<td>$2 \frac{1}{2}$</td>
</tr>
<tr>
<td>Picture frame</td>
<td>$\frac{1}{4}$</td>
</tr>
</tbody>
</table>
What do you notice?
What do you wonder?

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</table>
Conversation Prompt: Use your Constructive Conversation Skills to interview your partner about their approach for solving the problem. Focus on Clarifying and Fortifying each other’s ideas.

A: How did you approach the problem?

B: My understanding is that Susan wants to make as many different art projects as possible with exactly four yards of ribbon. Therefore, I used a tape diagram to represent the total amount of ribbon she has. Next, I drew another bar model underneath and decomposed it into each individual yard. Finally, I started to subtract art projects from that total amount. Does that make sense?

A: What I understood was that you used fraction strips to build a tape diagram that represents the four yards of ribbon as whole and another that represents the yards for the items Susan might decide to make. Then you subtracted. I notice that you also drew a number line underneath your bar models. Can you elaborate on that idea?

B: My intention was to illustrate how the three projects combine to equal 4 yards. For example, the two orange fraction strips represent the two yards needed to make the costume, while the green strips represent the one and three-fourths of a yard required to make the flower. Is that clear?

A: Following the same logic, the yellow fraction strip represents the one-fourth of a yard needed for the picture frame. Aside from using a tape diagram and a number line, I noticed that you also used equations to show your thinking. Can you tell me what each number represents?

B: Of course. The numbers in the first equation represent the two yards for the costume project that I subtracted from the four yards of ribbon. As you can see, the difference was two yards. The next equation, I subtracted one and three-quarter yards for the flower project from the left over two yards of ribbon, which left me with one-fourth of a yard. The only art project left that uses one-fourth of a yard is the picture frame, so I ended up using all four yards of ribbon. What other questions might you have about my thinking?

A: I heard you say that your equations show how you subtracted the different art projects from the four yards of ribbon and I would like to add that your equations help me visualize what you did with your fraction bars, as you subtracted. How do you know your thinking addresses the problem?

B: I know that Susan wants to make as many different art projects as possible. I also know that she is using exactly four yards of ribbon. What I needed to find out is how many different art projects are possible Susan can make with exactly four yards of ribbon. So I figured I’ll just subtract the different art projects that used up exactly four yards and came up with three different art projects. How does my idea compare to yours?
Prompt Starters:

• How did you approach the problem?
• Can you elaborate on that idea?
• Why did you…?
• How do you know your thinking makes sense?
• What made you think that…?

Response Starters:

• To solve the problem, I initially...
• For my next step… therefore…
• Afterward… because…
• Next… Finally, …
• For example, … Is that clear?
• Specifically, I used… to represent… Does that make sense?
• What other questions do you have about my thinking?