Mathematics Claim #3

COMMUNICATING REASONING

Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

Rationale for Claim #3

This claim refers to a recurring theme in the CCSSM content and practice standards: the ability to construct and present a clear, logical, convincing argument. For older students this may take the form of a rigorous deductive proof based on clearly stated axioms. For younger students this will involve more informal justifications. Assessment tasks that address this claim will typically present a claim or a proposed solution to a problem and will ask students to provide, for example, a justification, and explanation, or counter-example.

Rigor in reasoning is about the precision and logical progression of an argument: first avoiding making false statements, then saying more precisely what one assumes, and providing the sequence of deductions one makes on this basis. Assessments for this claim should use tasks that examine a student's ability to analyze a provided explanation, to identify flaws, to present a logical sequence, and to arrive at a correct argument.

> "Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments." (Practice 3, CCSSM)

Items and tasks supporting this claim should also assess a student's proficiency in using concepts and definitions in their explanations:

> "Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions." (Practice 6, CCSSM)

What sufficient evidence looks like for Claim #3

Assessment of this claim can be accomplished with a variety of item/task types, including selected response and short constructed response items, and with extended constructed response tasks. Sufficient evidence would be unlikely to be produced if students were not expected to produce communications about their own reasoning and the reasoning of others. That said, students are likely to be unfamiliar with assessment tasks asking them to explain their reasoning. In order to develop items/tasks that capture student reasoning, it will be important for early piloting and cognitive labs to explore and understand how students express their explanations of reasoning. As students (and teachers) become more familiar with the expectations of the assessment, and as instruction in the Common Core takes hold, students will become more and more successful on tasks aligned to Claim #3 with increasing frequency.

Items and tasks aligned to this claim should reflect the values set out for this claim, giving substantial weight to the quality and precision of the reasoning reflected in at least one, or several of the manners listed below. Options for selected response items and scoring guides for constructed response tasks should be developed to provide credit for demonstration of reasoning and to capture and identify flaws in student logic or reasoning. Features of options and scoring guides include:

- Assuring an explanation of the assumptions made;
- Asking for or recognizing the construction of conjectures that appear plausible, where appropriate;
- Having the student construct examples (or asking the student to distinguish among appropriate and inappropriate examples) in order to evaluate the proposition or conjecture;
- Requiring the student to describe or identify flaws or gaps in an argument;
- Evaluating the clarity and precision with which the student constructs a logical sequence of steps to show how the assumptions lead to the acceptance or refutation of a proposition or conjecture;
- Rating the precision with which the student describes the domain of validity of the proposition or conjecture.

As noted above, communicating mathematical reasoning is not just a requirement of the Standards for Mathematical Practice—it is also a recurrent theme in the Standards for Mathematical Content. For example, many content standards call for students to explain, justify, or illustrate. Below is content standard 4.NBT.5—note the highlighted words:

> Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

The Smarter/Balanced assessments will attend thoroughly to those places in the content standards that call explicitly for communicating mathematical reasoning. This is important so that the system captures the Standards' evident design for "doing content differently" at these important junctures. Students are not asked to "Reason" in the abstract—rather, they are asked to reason about the central ideas in mathematics that they are studying. This is an important element of making mathematics education coherent for students. Clearly, the reasoning elements of the content standards cannot be thoroughly assessed under Claim #1 alone. Therefore, in order to measure the full range of the Standards, Claim #3 tasks must be used to assess those parts of the content standards that call for communicating reasoning. In practice, this implies that the large majority of Claim #3 tasks, at least 70%, will be written at small grain size, keyed primarily to a single content standard or part thereof which concerns communicating mathematical reasoning. Targeted content standards for Claim #3 will always belong to the major work of the grade (as in the 4.NBT.5 example shown above). These features justify the weight of Claim #3 in the summative score even as they ensure that Claim #3 actively promotes both the focus and coherence of the Standards.

Occasionally, Claim #3 items/tasks may involve the application of concepts and procedures across more than one content domain. Because of the high strategic demand that such substantial non-routine tasks present, the technical demand for these items/tasks will be lower – typically met by content first taught in earlier grades, consistent with the emphases described under Claim #1.

Accessibility and Claim #3: Successful performance under Claim #3 requires a high level of linguistic proficiency. Many students with disabilities have difficulty with written expression, whether via putting pencil to paper or fingers to computer. The claim does not suggest that correct spelling or punctuation is a critical part of the construction of a viable argument, nor does it suggest that the argument has to be in words. Thus, for those students whose disabilities create barriers to development of text for demonstrating reasoning and formation of an argument, it is appropriate to model an argument via symbols, geometric shapes, or calculator or computer graphic programs. As for Claims #1 and #2, access via text to speech and expression via scribe, computer, or speech to text technology will be important avenues for enabling many students with disabilities to construct viable arguments.

The extensive communication skills anticipated by this claim may also be challenging for many ELL students who nonetheless have mastered the content. Thus it will be important to provide multiple opportunities to ELL students for explaining their ideas through different methods and at different levels of linguistic complexity. Based on the data on ELL students' level of proficiency in L1 and L2, it will be useful to provide opportunities as appropriate for bilingual explanations of the outcomes. Furthermore, students' engagement in critique and debate should not be limited to oral or written words, but can be demonstrated through diagrams, tables, and structured mathematical responses where students provide examples or counter-examples of additional problems.

Assessment Targets for Claim #3

Claim #3 is aligned to the mathematical practices from the MCCSS. For this reason, the Assessment Targets are all *acts of reasoning* that are consistent across grades and also evolve across grades. Consistent with the above discussion, these acts of reasoning are also tied to content (CCSSM, p. 8).

SUMMATIVE ASSESSMENT TARGETS **Providing Evidence Supporting Claim #3**
Claim #3: Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.
To preserve the focus and coherence of the standards as a whole, tasks must draw clearly on knowledge and skills that are articulated in the content standards. At each grade level, the content standards offer natural and productive settings for generating evidence for Claim #3. Tasks generating evidence for Claim #3 in a given grade will draw upon knowledge and skills articulated in the standards in that same grade, with strong emphasis on the major work of the grade. Any given task will provide evidence for several of the following assessment targets; each of the following targets should not lead to a separate task.

Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8	High School	
3.OA.B	4.OA.3	5.NBT.2	6.RP.A	7.RP.2	8.EE.1	N-RN.A	G-CO.A
3.NF.A	4.NBT.A	5.NBT.6	6.RP.3	7.NS.A	8.EE.5	N-RN.B	G-CO.B
3.NF.1	4.NBT.5	5.NBT.7	6.NS.A	7.NS.1	8.EE.6	N-RN.3	G-CO.C
3.NF.2	4.NBT.6	5.NF.1	6.NS.1	7.NS.2	8.EE.7a	A-SSE.2	G-CO.9
3.NF.3	4.NF.A	5.NF.2	6.NS.C	7.EE.1	8.EE.7b	A-APR.1	G-CO.10
3.MD.A	4.NF.1	5.NF.B	6.NS.5	7.EE.2	8.EE.8a	A-APR.B	G-CO.11
3.MD.7	4.NF.2	5.NF.3	6.NS.6		8.F.1	A-APR.4	G.SRT.A
	4.NF.3a	5.NF.4	6.NS.7		8.F.2	A-APR.6	G.SRT.B
	4.NF.3b	5.NF.7a	6.EE.A		8.F.3	A-REI.A	F-TF.1
	4.NF.3c	5.NF.7b	6.EE.3		8.G.1	A-REI.1	F-TF.2
	4.NF.4a	5.MD.C	6.EE.4		8.G.2	A-REI.2	F-TF.8
	4.NF.4b	5.MD.5a	6.EE.B		8.G.4	A-REI.C	
	4.NF.C	5.MD.5b	6.EE.6		8.G.5	A-REI.10	
	4.NF.7	5.G.B*	6.EE.9		8.G.6	A-REI.11	
		5.G.4*			8.G.8	F-IF.1	
						F-IF.5	
						F-IF.9	
						F-BF.3	
						F-BF.4a	

*Denotes additional and supporting clusters

Target A: Test propositions or conjectures with specific examples. (DOK 2)

Target B: Construct, autonomously,[12] chains of reasoning that will justify or refute propositions or conjectures. (DOK 3, 4).[13]

Target C: State logical assumptions being used. (DOK 2, 3)

Target D: Use the technique of breaking an argument into cases. (DOK 2, 3)

Target E: Distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in the argument—explain what it is. (DOK 2, 3, 4)

Target F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions. (DOK 2, 3)

Target G: At later grades, determine conditions under which an argument does and does not apply. (For example, area increases with perimeter for squares, but not for all plane figures.) (DOK 3, 4)

[12] By "autonomous" we mean that the student responds to a single prompt, without further guidance within the task.

[13] At the secondary level, these chains may take a successful student 10 minutes to construct and explain. Times will be somewhat shorter for younger students, but still giving them time to think and explain. For a minority of these tasks, subtasks may be constructed to facilitate entry and assess student progress towards expertise. Even for such "apprentice tasks" part of the task will involve a chain of autonomous reasoning that takes at least 5 minutes.

Types of Extended Response Tasks for Claim #3

Proof and justification tasks: These begin with a proposition and the task is to provide a reasoned argument why the proposition is or is not true. In other tasks, students may be asked to characterize the domain for which the proposition is true (see Assessment Target G).

Example of a standard proof task

Math – Grade 11	**Item Type: CR**	**DOK: (Webb 1- 4) 3**

Domain(s): Geometry

Content Cluster(s) and/or Standard(s):

G.CO **Prove** geometric theorems

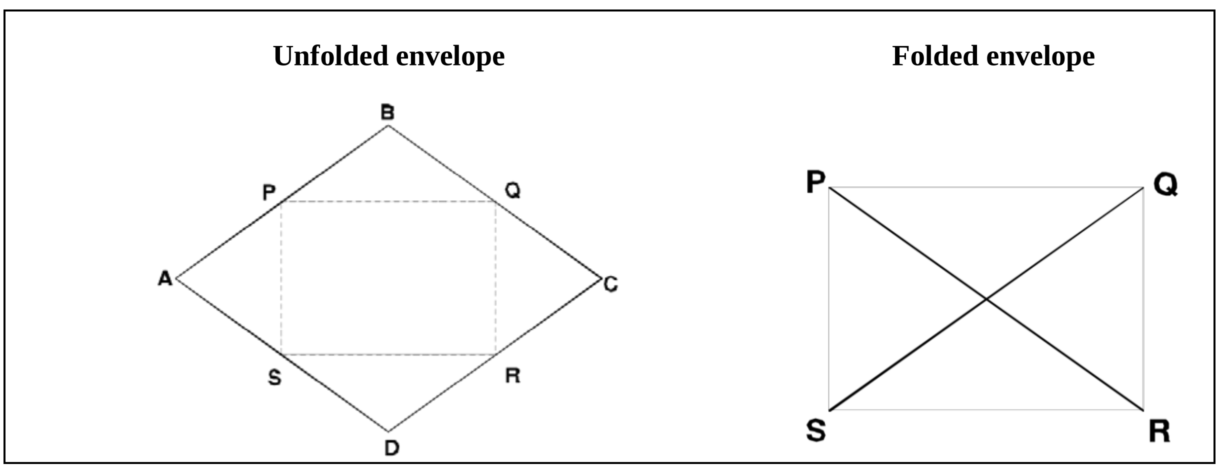
G.CO.11 **Prove** theorems about parallelograms.

Claim #3 Assessment Targets
Target B: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures.

Target C: State logical assumptions being used.

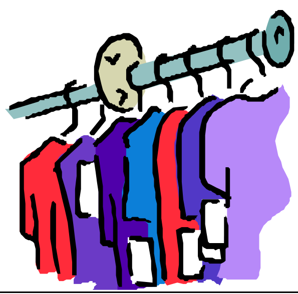
Target F: Base arguments on concrete referents such as objects, drawings, diagrams, and actions.

The Envelope



Prove that when the rectangular envelope (PQRS) is unfolded, the shape obtained (ABCD) is a rhombus.

Critiquing tasks: Some flawed 'student' reasoning is presented and the task is to correct and improve it.

Math – Grade 7	**Item Type: CR**	**DOK: (Webb 1- 4) 3**
Domain(s): Ratios and Proportional Relationships **Content Cluster(s) and/or Standard(s)** 7.RP **Analyze** proportional relationships and use them to **solve** real-world and mathematical problems. 7.RP.3 Use proportional relationships to **solve** multistep ratio and percent problems.		
Claim #3 Assessment Targets Target A: Test propositions or conjectures with specific examples. Target B: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures. Target D: Use the technique of breaking an argument into cases. Target E: Distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in the argument, explain what it is.		
Sale prices Max bought 2 items in a sale. One item was 10% off. One item was 20% off. Max says he saved 15% altogether. Is he right? Explain. 		

Mathematical investigations: Students are presented with a phenomenon and are invited to formulate conjectures about it. They are then asked to go on and prove one of their conjectures. This kind of task benefits from a longer time scale, and might best be incorporated into items/tasks associated with the Performance Tasks that afford a longer period of time for students to complete their work.

> ***Sums of Consecutive Numbers***
>
> Many whole numbers can be expressed as the sum of two or more positive consecutive whole numbers, some of them in more than one way.
>
> For example, the number 5 can be written as
>
> 5 = 2 + 3
>
> and that's the only way it can be written as a sum of consecutive whole numbers.
>
> In contrast, the number 15 can be written as the sum of consecutive whole numbers in three different ways:
>
> 15 = 7 + 8
>
> 15 = 4 + 5 + 6
>
> 15 = 1 + 2 + 3 + 4 + 5
>
> Now look at other numbers and find out all you can about writing them as sums of consecutive whole numbers.
>
> Write an account of your investigation. If you find any patterns in your results, be sure to point them out, and also try to explain them fully.

This is not a complete list; other types of task that fit the criteria above may well be included. But a balanced mixture of these types will provide enough evidence for Claim #3.