Mathematics Claim #1

CONCEPTS AND PROCEDURES

Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.

Rationale for Claim #1

This claim addresses procedural skills and the conceptual understanding on which developing skills depend. It is important to assess how aware students are of how concepts link together, and why mathematical procedures work in the way that they do. This relates to the structural nature of mathematics:

> Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. (Practice 7, CCSSM)
>
> They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. (Practice 7, CCSSM)
>
> Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results. (Practice 8, CCSSM)

Assessments should include items/tasks that test the precision with which students are able to carry out procedures, describe concepts and communicate results.

> Mathematically proficient students … state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of

> measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. (Practice 6, CCSSM)

Items/tasks should also assess how well students are able to use appropriate tools strategically.

> Students are able to use technological tools to explore and deepen their understanding of concepts. (Practice 5; CCSSM)

Many individual content standards in CCSSM set an expectation that students can *explain why* given procedures work.

> One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness. (CCSSM, p.4).

Finally, throughout the K-6 standards in CCSSM there are also individual content standards that set expectations for fluency in computation (e.g., fluent multiplication and division within the times tables in Grade 3). Such standards are culminations of progressions of learning, often spanning several grades, involving conceptual understanding, thoughtful practice, and extra support where necessary. Technology may offer the promise of assessing fluency more thoughtfully than has been done in the past. This, too, is part of 'measuring the full range of the standards.'

Following our discussion of the types of evidence appropriate for contributing to assessment of Claim #1, we describe specific grade-level content emphases.

What sufficient evidence looks like for Claim #1

Evidence on each student's progress along the progressions of mathematical content is the focus of attention in assessing this claim.

Essential properties of items and tasks that assess this claim: Items and tasks that could provide evidence for this claim include brief items – selected response and short constructed response items – that focus on a particular procedural skill or concept. Brief items could also include items that require students to translate between or among representations of concepts (words, diagrams, symbols) and items that require students to identify an underlying structure. Brief constructed response items can include items that provide scaffolded support for the student; it is probably possible for a Computer Adaptive environment to adjust the level of scaffolding that is provided depending on the student's performance level.

Selected response items, including computer-enhanced items, can probe conceptual understanding, particularly when the distractors are chosen to embody common misconceptions. In designing such items, it is essential to try to make sure that students do not obtain correct answers because of "test taking skills" rather than understanding of the mathematical content. Computer administration of the assessment affords the possibility of assessing student fluency with mathematical operations by means of monitoring the response time.

Short Constructed response items can assess mathematical thinking directly; short items of this kind can provide direct evidence of students' mastery of standard procedures. Among items/tasks that require students to produce a response, short constructed response items are the most likely to be able to be machine scored.

Highly scaffolded tasks, where the student is guided through a series of short steps set in a common problem context, offer another approach to the design of short constructed response items.

Extended Response items, requiring a more solid demonstration of conceptual understanding and procedural skills that students may be expected to have learned and practiced, may also provide evidence for this claim. These can include the following task types**:**

- **Application tasks** using exercises to assess relatively standard applications of mathematical principles. Here, students can be expected to use important concepts and skills to tackle problem situations that should be in the learned part of the curriculum.
- **Translation tasks**, where students are asked to represent concepts in different ways and translate between representations (words, numbers, tables, graphs, symbolic algebra).
- **Explanation tasks,** where students are asked to explain why a given standard procedure works. This may involve the straightforward adaptation of a standard procedure.

Accessibility & Claim #1: This claim clarifies the importance of conceptual understanding and procedural knowledge underlying the important core content in CCSSM. The standards refer to the ability to carry out procedures, describe concepts, communicate results, use appropriate tools strategically, and explain why specific procedures make sense. Neither the claim itself nor the CCSSM explicitly addresses the challenges that some students with disabilities face in the area of mathematical calculations. Because of the importance of building skills in computation in early schooling, the explication of the content may be different in early school grades compared to later school grades. Providing assistive technologies such as an abacus or calculator may not be considered appropriate up through about grade 4. At some point during intermediate grades, however, the use of these tools is considered an appropriate avenue of access to allow students to demonstrate that they are able to "calculate accurately and efficiently."

It is also important to address access to mathematics via decoding text and written expression. The uses of alternative means of access and expression are ones used by successful individuals (Reitz, 2011) to demonstrate high levels of success, and thus are an appropriate avenue of access to the content for students with disabilities in the areas of reading decoding and fluency as well as for those with blindness

or visual impairments. Likewise, allowing students alternative ways to express their understanding of mathematics content is important. Students who are unable to explain mathematical processes via writing or computer entry might instead provide their explanation via speech to text technology (or a scribe) or via manipulation of physical objects.

A major aspect of all the claims, including Claim #1, is communication, especially students' ability to explain *why or how* given procedures or approaches work. To maximize access to English learners who are at a lower proficiency in writing and speaking, it is important for Smarter Balanced to explore allowing ELL students to use diagrams, drawings, equations, and mathematical models, as well as words. It will also be useful to provide opportunities for ELL students to communicate their understanding through performance tasks or other approaches where multiple domain input can be provided. Furthermore, when a major performance difference exists between tasks such as expanding and explaining, it will be important to allow students to express their views through the use of native language, where that is appropriate.

Assessment Targets for Claim #1

Cluster headings as assessment targets: In the CCSSM the cluster headings usually serve to communicate the larger intent of a group of standards. For example, a cluster heading in Grade 4 reads: "Generalize understanding of place value for multi-digit numbers." Individual standards in this cluster pinpoint some signs of success in the endeavor, but the important endeavor itself is stated directly in the cluster heading. In addition, the word "generalize" signals that there is a multi-grade progression in grades K-3 leading up to this group of standards. With this in mind, the cluster headings can be viewed as the most effective means of communicating the focus and coherence of the standards. Therefore, this content specifications document *uses the cluster headings as the targets of assessment* for generating evidence for Claim #1. For each cluster, guidance is provided that gives item developers important information about item/task considerations for the cluster. Sample items are also provided that illustrate the content scope and range of difficulty appropriate to assess a cluster. Claim #1 assessment targets are shown below for Grades 3 through 8 and Grade 11. Content emphases for all grades are shown in the tables for Claim 1, which are based on the cluster level of the Common Core State Standards for Mathematics.

Content emphases in the standards: In keeping with the design principles of focus and coherence in the standards as a whole, not all content is emphasized equally in the Standards for Mathematical Content.

- The standards communicate emphases in many ways, including by the use of domain names that vary across the grades, and that are sometimes much more fine-grained than the top-level

organizers in previous state standards (e.g., Ratios and Proportional Relationships). These and other features of the standards and their progressions point to the major work of each grade.[4]

- Standards for topics that are not major emphases in themselves are generally written in such a way as to support and strengthen the areas of major emphases. This promotes valuable connections that add coherence to the grade. Still other topics that may not connect tightly or explicitly to the major work of the grade would fairly be called additional.

In the tables that follow, these designations—"major" and "additional/supporting" — are provided at the cluster level.

Working at the cluster level helps to avoid obscuring the big ideas and getting lost in the details of specific standards (which are individually important, but impossible to measure in their entirety within the bounds of reasonable testing time). Clusters provide an appropriate grain size for following the contours of important progressions in the standards across grades, for example: the integration of place value understanding and the meanings and properties of operations that must happen as students develop computation strategies and algorithms for multi-digit numbers during grades K-6; or the appropriate development of functional thinking in middle school leading to the emergence of functions as a content domain in Grade 8.

Identifying some standards within "major" clusters and others within "additional/supporting" clusters is not to say that anything in the standards can be neglected. To do so would leave gaps in student preparation for later mathematics. In other words, all content is eligible for and should be encompassed in the assessment. However, evidence for Claim #1 will strongly focus on the major clusters and take into account ways in which the standards tie supporting clusters to the major work of each grade, such that the items/tasks seen by every student will sample in much greater proportion from clusters representing the major work of each grade. Appendix A provides a sampling scheme for the CAT engine that reflects the structure of the standards and captures emphases appropriately at each grade.

In what follows, Claim #1 Assessment Targets are provided for grades 3 through 8 and high school.

[4] Further detail on emphases can be seen in the *Progressions* documents drafted by members of the Common Core State Standards Working Group, and published through the Institute for Mathematics and Education of the University of Arizona: http://ime.math.arizona.edu/progressions/. More information is also available in the *K-8 Publishers' Criteria*, developed by the CCSSM authors, available at www.corestandards.org.

GRADE 3 Summative Assessment Targets Providing Evidence Supporting Claim #1
Claim #1: Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Content for this claim may be drawn from any of the Grade 3 clusters represented below, with a much greater proportion drawn from clusters designated "m" (major) and the remainder drawn from clusters designated "a/s" (additional/supporting) – with these items fleshing out the major work of the grade. Sampling of Claim #1 assessment targets will be determined by balancing the content assessed with items and tasks for Claims #2, #3, and #4. Detailed information about how each Claim 1 assessment target is measured can be found in the Item Specifications "Mathematics Grades 3-5" zip folder available at http://www.smarterbalanced.org/smarter-balanced-assessments/.
Operations and Algebraic Thinking
Target A [m]: Represent and solve problems involving multiplication and division.[5] (DOK 1)
Target B [m]: Understand properties of multiplication and the relationship between multiplication and division. (DOK 1)
Target C [m]: Multiply and divide within 100. (DOK 1)
Target D [m]: Solve problems involving the four operations, and identify and explain patterns in arithmetic. (DOK 2)
Number and Operations—Base Ten
Target E [a/s]: Use place value understanding and properties of arithmetic to perform multi-digit arithmetic. (DOK 1)
Number and Operations—Fractions
Target F [m]: Develop understanding of fractions as numbers. (DOK 1, 2)
Measurement and Data
Target G [m]: Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects. (DOK 1, 2)
Target H [a/s]: Represent and interpret data. (DOK 2)
Target I [m]: Geometric measurement: understand concepts of area and relate area to multiplication and to addition. (DOK 2)
Target J [a/s]: Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures. (DOK 1)
Geometry
Target K [a/s]: Reason with shapes and their attributes. (DOK 1, 2)

[5] See CCSSM, Table 2, p. 89 for additional information.

Grade 4 SUMMATIVE ASSESSMENT TARGETS Providing Evidence Supporting Claim #1
Claim #1: Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Content for this claim may be drawn from any of the Grade 4 clusters represented below, with a much greater proportion drawn from clusters designated "m" (major) and the remainder drawn from clusters designated "a/s" (additional/supporting) – with these items fleshing out the major work of the grade. Sampling of Claim #1 assessment targets will be determined by balancing the content assessed with items and tasks for Claims #2, #3, and #4. Detailed information about how each Claim 1 assessment target is measured can be found in the Item Specifications "Mathematics Grades 3-5" zip folder available at http://www.smarterbalanced.org/smarter-balanced-assessments/.
Operations and Algebraic Thinking (4.OA)
Target A [m]: Use the four operations with whole numbers to solve problems. (DOK 1, 2)
Target B [a/s]: Gain familiarity with factors and multiples. (DOK 1)
Target C [a/s]: Generate and analyze patterns. (DOK 2, 3)
Number and Operations in Base Ten (4.NBT)
Target D [m]: Generalize place value understanding for multi-digit whole numbers. (DOK 1, 2)
Target E [m]: Use place value understanding and properties of operations to perform multi-digit arithmetic. (DOK 1, 2)
Number and Operations – Fractions (4.NF)
Target F [m]: Extend understanding of fraction equivalence and ordering. (DOK 1, 2)
Target G [m]: Build fractions from unit fractions by applying and extending previous understandings of **Target H [m]: Understand decimal notation for fractions, and compare decimal fractions. (DOK 1, 2)**
Measurement and Data (4.MD)
Target I [a/s]: Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. (DOK 1, 2)
Target J [a/s]: Represent and interpret data. (DOK 1, 2) **Target K [a/s]: Geometric measurement: understand concepts of angle and measure angles. (DOK 1, 2)**
Geometry (4.G)
Target L [a/s]: Draw and identify lines and angles, and classify shapes by properties of their lines and angles. (DOK 1, 2)

Grade 5 SUMMATIVE ASSESSMENT TARGETS Providing Evidence Supporting Claim #1
Claim #1: Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Content for this claim may be drawn from any of the Grade 5 clusters represented below, with a much greater proportion drawn from clusters designated "m" (major) and the remainder drawn from clusters designated "a/s" (additional/supporting) – with these items fleshing out the major work of the grade. Sampling of Claim #1 assessment targets will be determined by balancing the content assessed with items and tasks for Claims #2, #3, and #4. Detailed information about how each Claim 1 assessment target is measured can be found in the Item Specifications "Mathematics Grades 3-5" zip folder available at http://www.smarterbalanced.org/smarter-balanced-assessments/.
Operations and Algebraic Thinking
Target A [a/s]: Write and interpret numerical expressions. (DOK 1)
Target B [a/s]: Analyze patterns and relationships. (DOK 2)
Number and Operations—Base Ten
Target C [m]: Understand the place value system. (DOK 1, 2)
Target D [m]: Perform operations with multi-digit whole numbers and with decimals to hundredths. (DOK 1, 2)
Number and Operations—Fractions
Target E [m]: Use equivalent fractions as a strategy to add and subtract fractions. (DOK 1, 2)
Target F [m]: Apply and extend previous understandings of multiplication and division to multiply and divide fractions. (DOK 1, 2)
Measurement and Data
Target G [a/s]: Convert like measurement units within a given measurement system. (DOK 1)
Target H [a/s]: Represent and interpret data. (DOK 2)
Target I [m]: Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition. (DOK 1, 2)
Geometry
Target J [a/s]: Graph points on the coordinate plane to solve real-world and mathematical problems. (DOK 1)
Target K [a/s]: Classify two-dimensional figures into categories based on their properties. (DOK 2)

Grade 6 SUMMATIVE ASSESSMENT TARGETS Providing Evidence Supporting Claim #1
Claim #1: Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Content for this claim may be drawn from any of the Grade 6 clusters represented below, with a much greater proportion drawn from clusters designated "m" (major) and the remainder drawn from clusters designated "a/s" (additional/supporting) – with these items fleshing out the major work of the grade. Sampling of Claim #1 assessment targets will be determined by balancing the content assessed with items and tasks for Claims #2, #3, and #4. Detailed information about how each Claim 1 assessment target is measured can be found in the Item Specifications "Mathematics Grades 6-8" zip folder available at http://www.smarterbalanced.org/smarter-balanced-assessments/.
Ratios and Proportional Relationships (6.RP)
Target A [m]: Understand ratio concepts and use ratio reasoning to solve problems. (DOK 1, 2)
The Number System (6.NS)
Target B [m]: Apply and extend previous understandings of multiplication and division to divide fractions by fractions. (DOK 1, 2) **Target C [a/s]: Compute fluently with multi-digit numbers and find common factors and multiples. (DOK 1, 2)** **Target D [m]: Apply and extend previous understandings of numbers to the system of rational numbers. (DOK 1, 2)**
Expressions and Equations (6.EE)
Target E [m]: Apply and extend previous understandings of arithmetic to algebraic expressions. (DOK 1, 2) **Target F [m]: Reason about and solve one-variable equations and inequalities. (DOK 1, 2)** **Target G [m]: Represent and analyze quantitative relationships between dependent and independent variables. (DOK 2)**
Geometry (6.G)
Target H [a/s]: Solve real-world and mathematical problems involving area, surface area, and volume. (DOK 1, 2)
Statistics and Probability (6.SP)
Target I [a/s]: Develop understanding of statistical variability. (DOK 2) **Target J [a/s]: Summarize and describe distributions. (DOK 1, 2)**

Grade 7 SUMMATIVE ASSESSMENT TARGETS Providing Evidence Supporting Claim #1
Claim #1: Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Content for this claim may be drawn from any of the Grade 7 clusters represented below, with a much greater proportion drawn from clusters designated "m" (major) and the remainder drawn from clusters designated "a/s" (additional/supporting) – with these items fleshing out the major work of the grade. Sampling of Claim #1 assessment targets will be determined by balancing the content assessed with items and tasks for Claims #2, #3, and #4. Detailed information about how each Claim 1 assessment target is measured can be found in the Item Specifications "Mathematics Grades 6-8" zip folder available at http://www.smarterbalanced.org/smarter-balanced-assessments/.
Ratios and Proportional Relationships (7.RP)
Target A [m]: Analyze proportional relationships and use them to solve real-world and mathematical problems. (DOK 2)
The Number System (7.NS)
Target B [m]: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers. (DOK 1, 2)
Expressions and Equations (7.EE)
Target C [m]: Use properties of operations to generate equivalent expressions. (DOK 1, 2)
Target D [m]: Solve real-life and mathematical problems using numerical and algebraic expressions and equations. (DOK 1, 2)
Geometry (7.G)
Target E [a/s]: Draw, construct and describe geometrical figures and describe the relationships between them. (DOK 1, 2)
Target F [a/s]: Solve real-life and mathematical problems involving angle measure, area, surface area, and volume. (DOK 1, 2)
Statistics and Probability (7.SP)
Target G [a/s]: Use random sampling to draw inferences about a population. (DOK 1, 2)
Target H [a/s]: Draw informal comparative inferences about two populations. (DOK 2)
Target I [a/s]: Investigate chance processes and develop, use, and evaluate probability models. (DOK 1, 2)

Grade 8 SUMMATIVE ASSESSMENT TARGETS Providing Evidence Supporting Claim #1
Claim #1: Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Content for this claim may be drawn from any of the Grade 8 clusters represented below, with a much greater proportion drawn from clusters designated "m" (major) and the remainder drawn from clusters designated "a/s" (additional/supporting) – with these items fleshing out the major work of the grade. Sampling of Claim #1 assessment targets will be determined by balancing the content assessed with items and tasks for Claims #2, #3, and #4. Detailed information about how each Claim 1 assessment target is measured can be found in the Item Specifications "Mathematics Grades 6-8" zip folder available at http://www.smarterbalanced.org/smarter-balanced-assessments/.
The Number System
Target A [a/s]: Know that there are numbers that are not rational, and approximate them by rational numbers. (DOK 1, 2)
Expressions and Equations
Target B [m]: Work with radicals and integer exponents. (DOK 1)
Target C [m] Understand the connections between proportional relationships, lines, and linear equations. (DOK 1, 2)
Target D [m]: Analyze and solve linear equations and pairs of simultaneous linear equations. (DOK 1, 2)
Functions
Target E [m]: Define, evaluate, and compare functions. (DOK 1, 2)
Target F [m]: Use functions to model relationships between quantities. (DOK 1, 2)
Geometry
Target G [m]: Understand congruence and similarity using physical models, transparencies, or geometry software. (DOK 1, 2)
Target H [m]: Understand and apply the Pythagorean theorem. (DOK 2)
Target I [a/s]: Solve real-world and mathematical problems involving volume of cylinders, cones and spheres. (DOK 2)
Statistics and Probability
Target J [a/s]: Investigate patterns of association in bivariate data. (DOK 1, 2)

Grade 11 SUMMATIVE ASSESSMENT TARGETS Providing Evidence Supporting Claim #1
Claim #1: Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.
Content for this claim may be drawn from any of the high school clusters represented below, with a much greater proportion drawn from clusters designated "m" (major) and the remainder drawn from clusters designated "a/s" (additional/supporting) – with these items fleshing out the major work of the grade. Sampling of Claim #1 assessment targets will be determined by balancing the content assessed with items and tasks for Claims #2, #3, and #4. Detailed information about how each Claim 1 assessment target is measured can be found in the Item Specifications "Mathematics High School" zip folder available at http://www.smarterbalanced.org/smarter-balanced-assessments/.
Number and Quantity (9-12.N)
Target A [a/s]: Extend the properties of exponents to rational exponents. (DOK 1, 2)
Target B [a/s]: Use properties of rational and irrational numbers. (DOK 1, 2)
Target C [m]: Reason quantitatively and use units to solve problems. (DOK 1, 2)
Algebra (9-12.A)
Target D [m]: Interpret the structure of expressions. (DOK 1)
Target E [m]: Write expressions in equivalent forms to solve problems. (DOK 1, 2)
Target F [a/s]: Perform arithmetic operations on polynomials. (DOK 1)
Target G [a/s]: Create equations that describe numbers or relationships. (DOK 1, 2)
Target H [m]: Understand solving equations as a process of reasoning and explain the reasoning. (DOK 1, 2)
Target I [m]: Solve equations and inequalities in one variable. (DOK 1, 2)
Target J [m]: Represent and solve equations and inequalities graphically. (DOK 1, 2)
Functions (9-12.F)
Target K [m]: Understand the concept of a function and use function notation. (DOK 1)
Target L [m]: Interpret functions that arise in applications in terms of a context. (DOK 1, 2)
Target M [m]: Analyze functions using different representations. (DOK 1, 2, 3)
Target N [m]: Build a function that models a relationship between two quantities. (DOK 1, 2)
Geometry (9-12.G)
Target O: Define trigonometric ratios and solve problems involving right triangles (DOK 1, 2)
Statistics and Probability (9-12.SP)
Target P [m]: Summarize, represent and interpret data on a single count or measurement variable. (DOK 2)

Notes on Grades 9-12 Content Clusters Not Identified as Assessment Targets for Claim 1

Algebra

Content from the remaining Algebra clusters will also provide content and context for tasks in Claims 2-4, though these will be sampled in lesser proportion than those explicitly listed as targets for Claim 1. Clusters not explicitly identified as targets for Claim 1 are the following:

- Understand the relationship between zeros and factors of polynomials
- Use polynomial identities to solve problems
- Rewrite rational expressions
- Solve systems of equations*

*Content from this cluster may be sampled in greater proportion due to its interconnectivity to some of the targets listed under Claim 1.

Functions

Content from the remaining Functions clusters will also provide content and context for tasks in Claims 2-4, though these will be sampled in lesser proportion than those explicitly listed as targets for Claim 1. Clusters not explicitly identified as targets for Claim 1 are the following:

- Build new functions from existing functions
- Construct and compare linear, quadratic, and exponential models and solve problems*
- Interpret expressions for functions in terms of the situation they model*
- Extend the domain of trigonometric functions using the unit circle
- Model periodic phenomena with trigonometric functions
- Prove and apply trigonometric identities

*Content from these clusters may be sampled in greater proportion due to its interconnectivity to some of the targets listed under Claim 1.

Geometry

While only one content cluster from the Geometry domain[6] is highlighted for task development under Claim 1, the remaining clusters will be used to build tasks for Claims 2-4. In general, the clusters listed below provide natural and productive opportunities to connect the work of algebra, functions and geometry in the context of problems for Claims 2-4:

- Use coordinates to prove simple geometric theorems algebraically
- Explain volume formulas and use them to solve problems
- Apply geometric concepts in modeling situations

Content from the remaining Geometry clusters will also provide content and context for tasks in Claims 2-4, though these will be sampled in lesser proportion than those listed above and that explicitly listed as a target for Claim 1.

- Experiment with transformations in the plane

[6] The phrase "Conceptual Category" is used in place of domain in the CCSS document. "Domain" is used here to maintain consistency with Grades 3-8 for the purposes of task development and item tagging.

- Understand congruence in terms of rigid motions
- Make geometric constructions
- Understand similarity in terms of similarity transformations
- Prove theorems involving similarity
- Prove geometric theorems
- Understand and apply theorems about circles
- Find arc lengths and areas of sectors of circles
- Translate between the geometric description and the equation for a conic section
- Visualize relationships between two-dimensional and three-dimensional objects

Statistics and Probability

While only one content cluster from the Statistics and Probability domain[7] is highlighted for task development under Claim 1, the remaining clusters will be used to build tasks for Claims 2-4. In general, the clusters listed below provide productive opportunities to connect the work of algebra, functions and statistics and probability in the context of problems for Claims 2-4:

- Summarize, represent, and interpret data on two categorical and quantitative variables
- Interpret linear models

Content from the remaining Statistics and Probability clusters will also provide content and context for tasks in Claims 2-4, though these will be sampled in lesser proportion than those listed above and that explicitly listed as a target for Claim 1.

- Understand and evaluate random processes underlying statistical experiments
- Make inferences and justify conclusions from sample surveys, experiments, and observational studies
- Understand independence and conditional probability and use them to interpret data
- Use the rules of probability to compute probabilities of compound events in a uniform probability model

Understanding Assessment Targets in an Adaptive Framework: In building an adaptive test, it is essential to understand how content gets "adapted." In a computer adaptive summative assessment, it doesn't make much sense to repeatedly offer formulaic multiplication and division items to a highly fluent Grade 3 student, making the Grade 3 Target OA.C [m] less relevant for this student than it may be for another. The higher-achieving student could be challenged further, while a student who is struggling could be given less complex items to ascertain how much each understands within the domain. The table below illustrates several items for the Grade 3 Operations and Algebraic Thinking domain that would likely span the difficulty spectrum for this grade. The items generally get more difficult with each row (an important feature of adaptive test item banks). (Pilot data will be used to determine more precisely the levels of difficulty associated with each kind of task.)

Sample for Grade 3, Claim #1 – Operations and Algebraic Thinking

[7] The phrase "Conceptual Category" is used in place of domain in the CCSS document. "Domain" is used here to maintain consistency with Grades 3-8 for the purposes of task development and item tagging.

Adapting Items within a Claim & Domain	Claim #1 – Operations and Algebraic Thinking
8 x 5 = □	Target C [m]: Multiply and divide within 100.
6 x □ = 30	Target A [m]: Represent and solve problems involving multiplication and division.
9 x 4 = □ x 9	Target B [m]: Understand properties of multiplication and the relationship between multiplication and division.
6 x 2 x □ = 60	Target B [m]: Understand properties of multiplication and the relationship between multiplication and division.
4 x 2 x □ = 5 x 2 x 2 x 2	Target B [m]: Understand properties of multiplication and the relationship between multiplication and division.
9 x 4 = 4 x □ x □ (May appear as a drag and drop TE item where "1" is not one of the choices for dragging.)	Target B [m]: Understand properties of multiplication and the relationship between multiplication and division.
8 x □ = 4 x □ Give two different pairs of numbers that could fill the boxes to make a true equation (selected response, drag and drop, or fill-in would work).	Target B [m]: Understand properties of multiplication and the relationship between multiplication and division.

Some of the more difficult items in the table incorporate several elements of this potential Grade 3 progression (fluency with multiplication → understanding the "unknown whole number" in a multiplication problem → applying properties of operations). Thus, a student who is consistently successful with items like the one in the final rows would not necessarily be assessed on items in previous rows within an adaptive test. In this way adaptive testing has the benefit of reduced test length while providing coverage of a broad scope of knowledge and skills. Adapting to greater and lesser difficulty levels than those illustrated in the table may require the use of items from other grades.

The relative impact of a student's ability or inability to "multiply and divide within 100" (Target C) would likely affect his/her performance on other clusters in the domain of Operations and Algebraic Thinking, thus serving as a baseline for much of the other content in this domain.

The sample items in the table illustrate another point – that the cluster level of the CCSS provides a suitable grain size for the development of a well-supplied item bank for computer adaptive testing. Item quality should not be compromised, particularly in an adaptive framework, by unnecessarily writing items at too fine a grain size. Since efficiency and appropriate item selection are optimized by

minimizing constraints on the adaptive test (Thompson & Weiss, 2011), it is critical to ensure that items provide an appropriate range of difficulty within each domain for Claim #1.

Again, CAT sampling proportions for Claim 1 are given in Appendix A.