Los Angeles Unified School District
Office of Curriculum, Instruction and School Support

COURSE DESCRIPTION

Common Core Geometry Tutorial Lab is designed to provide foundational knowledge and intervention for students enrolled in or preparing to enroll in *Common Core Geometry*. This course serves not only as intervention, but also as support for students experiencing difficulty in mastering the core standards and academic language constraints of the *Common Core Geometry* course. *Common Core Geometry Tutorial Lab* is an elective mathematics course provided to students as a supplemental course to enhance the student's knowledge of prerequisite skills and academic language that is required in order to successfully access the standards-based *Common Core Geometry* course.

COURSE SYLLABUS

The standards for this intervention course are taken primarily from the *Common Core Grade 7* and *Common Core Grade 8* math standards and support the major clusters defined in the LAUSD Curricular Maps for *Common Core Geometry*. Additionally, **an immense element of this intervention course is an emphasis on student engagement with the Standards for Mathematical Practice on a daily basis.** The structure of this course is divided into four separate, but coherent, units mirroring the *Common Core Geometry* course. The aim of this intervention course is to support *Common Core Geometry* and to provide explicit, systematic, and intensive instruction for at-risk populations. As teachers strive to assist struggling students to reach the Common Core State Standards' expectations, they must be able to accurately identify areas of student deficit and match students to an appropriate academic intervention plan. An expectation from the *Common Core Geometry Tutorial Lab* is to create evidence-based intervention plans that are customized to individual students, and that are also tied to specific Common Core Standards.

Students enrolled in this intervention course **need to be assessed** on an ongoing basis to determine their needs for support and intervention. Teachers are encouraged to adapt their instruction through ongoing formative assessments to provide genuine, differentiated instruction. The outcome of the initial and ongoing assessments are to analyze and identify key skills and concepts required for students to access the Common Core State Standards, compare those requirements to the student's existing skill set, and analyze any potential student deficits.

According to the California CCSS Mathematics Framework (November, 2013),

> "Universal Access in education is a concept which utilizes strategies for planning for the widest variety of learners from the beginning of the lesson design and not 'added on' as an afterthought. Universal Access is not a set of curriculum materials or specific time set aside for additional assistance but rather a schema. For students to benefit from universal access, teachers may need assistance in planning instruction, differentiating curriculum, infusing Specially Designed Academic Instruction in English (SDAIE) techniques, using the California English Language Development Standards (CA ELD standards), and using grouping strategies effectively."

Therefore, through careful planning for modifying curriculum, instruction, grouping, and assessment techniques, teachers are well prepared to adapt instruction to meet the needs of diverse learners in their classrooms.

Multi-tier Mathematics Interventions

Gersten et. al. (2009) in the Practice Guide "Assisting Students Struggling with Mathematics: RtI for Elementary and Middle School" presented evidence for the effectiveness of combinations of systematic and explicit instruction that include teacher demonstrations and think alouds early in the lesson, unit, or module; student verbalization of how a problem was solved; scaffolded practice; and immediate corrective feedback. In instruction that is systematic, concepts are introduced in a logical, coherent order and students have many opportunities to apply each concept. Below are the recommendations (Recommendations 3 and 4 received strong evidence rating).

Recommendation 1. Screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk. *It is suggested that you use any of the following instruments to screen students: MDTP, Scholastic Math Inventory, Easy CMB, etc.*

Recommendation 2. Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in kindergarten through grade 5 and on rational numbers in grades 4 through 8. These materials should be selected by committee.

Recommendation 3. Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.

Recommendation 4. Interventions should include instruction on solving word problems that is based on common underlying structures. *Teachers may consider using some of the strategies in "Improving Mathematical Problem Solving in Grades 4 Through 8" in teaching students problem solving*.

Recommendation 5. Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.

Recommendation 6. Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.

Recommendation 7. Monitor the progress of students receiving supplemental instruction and other students who are at risk.

Unit 1
Congruence through Transformations

Concepts/Clusters	Standards to Support CC Geometry	Suggested Resources
Make geometric constructions	**7.G.2:** Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	1. ***Math Open Reference***: Online construction demonstrations. http://mathopenref.com/tocs/constructionstoc.html 2. ***Engage NY***: Congruence, Proof, Constructions. http://www.engageny.org/resource/geometry-module-1 3. ***NCTM Illuminations***: Maya Constructions. http://illuminations.nctm.org/Lesson.aspx?id=4004
Experiment with transformations in the plane	**8.G.1:** Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.	1. ***MARS Task***: Representing and Combining Transformations. http://map.mathshell.org/materials/lessons.php?taskid=490#task490 2. ***Engage NY***: The Concept of Congruence. http://www.engageny.org/resource/grade-8-mathematics-module-2 3. ***NCTM Illuminations***: Cyclic Figures. http://illuminations.nctm.org/Activity.aspx?id=4097 4. ***NCTM Illuminations:*** Dihedral Figures. http://illuminations.nctm.org/Activity.aspx?id=4100 5. ***Inside Mathematics***: "Cut It Out" activity. http://www.insidemathematics.org/problems-of-the-month/pom-cutitout.pdf 6. ***MARS Task***: Aaron's Designs. http://map.mathshell.org/materials/tasks.php?taskid=361 7. ***Newark Public Schools***: Similarity and Congruence Module. http://bit.ly/newarkmodule
Understand congruence in terms of rigid motions	**8.G.2:** Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	1. ***Illustrative Mathematics***: Congruent Segments. https://www.illustrativemathematics.org/illustrations/646 2. ***Illustrative Mathematics***: Congruent Rectangles. https://www.illustrativemathematics.org/illustrations/1228 3. ***Illustrative Mathematics***: Congruent Triangles. https://www.illustrativemathematics.org/illustrations/1231

Concepts/Clusters	Standards to Support CC Geometry	Suggested Resources
Prove geometric theorems	**8.G.3:** Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	1. ***Illustrative Mathematics***: Reflecting Reflections. https://www.illustrativemathematics.org/illustrations/1243 2. ***Illustrative Mathematics***: Triangle Congruence with Coordinates. https://www.illustrativemathematics.org/illustrations/1232

Examples of Essential Questions for the Unit	Standards for Mathematical Practice
1. Why is it important to draw a figure using accurate conditions? 2. How are geometric attributes of a shape affected when movement occurs? 3. What tools can be used to make geometric constructions? 4. What types of transformations can be done in a plane? 5. How can we use rigid motions to understand congruence? 6. Distinguish the difference between transformations that are rigid (preserve distance and angle measure) and those that are not (dilations). 7. How is visualization essential to the study of geometry? 8. How might the concept of rigid motion connect to the concept of congruence? 9. How might a figure be transformed to map directly onto itself? Does order matter?	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Performance Objectives for Unit 1 *Students will grow in their ability to:*	Guiding Questions for Implementing Standards for Mathematical Practices #1 and #2
1. Make geometric constructions. 2. Visually represent geometric figures. 3. Understand the characteristics of angles (and sides) that create triangles. 4. Construct triangles from three given side measures to determine when there is a unique triangle, more than one triangle, or no triangle. 5. Experiment with transformations in the plane. 6. Understand that transformations produce images of exactly the same size and shape as the pre-image. 7. Verify that congruence of angles are maintained through rotations,	1. How might you describe this problem in your own words? 2. What are some other problems that are similar to this problem? 3. What do you notice about …? 4. What information is given in the problem? 5. Share with me the steps you've used up to this point. 6. What are some other strategies you might try? 7. Which steps in the process are you confident about? 8. Describe what you have already tried. What might you change?

Performance Objectives for Unit 1 *Students will grow in their ability to:*	**Guiding Questions for Implementing** **Standards for Mathematical Practices #1 and #2**
reflections, and translations. 8. Verify that when parallel lines are rotated, reflected or translated (each in the same way), they remain parallel lines. 9. Use physical models, transparencies, patty paper, or geometry software to verify the properties of transformations. 10. Understand congruence in terms of rigid motions. 11. Examine figures to determine congruency by identifying a sequence of rigid transformations that map one figure directly onto the other. 12. Apply the concept of congruency to write statements of congruency 13. Prove geometric theorems. 14. Understand that dilations are not forms of rigid transformations. 15. Understand that dilations either enlarge (if the scale factor is more than 1) or reduce (if the scale factor is less than 1) the size of a figure.	9. Describe the relationship between the two figures. 10. How is … related to …? 11. What is the relationship between … and …? 12. What properties might we use to find a solution? 13. How did you come to the decision that you needed to use …? 14. What might the numbers used in the problem represent? 15. What does this (figure, symbol, quantity, etc.) mean to you?

Unit 2
Similarity, Transformations, and Proofs

Concepts/Clusters	**Standards to Support CC Geometry**	**Suggested Resources**
Understand similarity in terms of similarity transformations	**8.G.4:** Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them. **7.G.1:** Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	1. ***Engage NY***: Similarity. http://www.engageny.org/resource/grade-8-mathematics-module-3 2. ***NCTM Illuminations***: In Your Shadow. http://illuminations.nctm.org/Lesson.aspx?id=1672 3. ***NCTM Illuminations***: Inversions. http://illuminations.nctm.org/Lesson.aspx?id=3979 4. ***MARS Task***: Photographs. http://map.mathshell.org/materials/download.php?fileid=1126

Concepts/Clusters	Standards to Support CC Geometry	Suggested Resources
Prove theorems involving similarity	**8.G.5:** Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*	1. ***MARS Task***: Finding Shortest Routes: The Schoolyard Problem. http://map.mathshell.org/materials/lessons.php?taskid=584#task584 2. ***MARS Task***: Identifying Similar Triangles. http://map.mathshell.org/materials/lessons.php?taskid=492#task492 ***3. Illustrative Mathematics***: Are They Similar? *https://www.illustrativemathematics.org/illustrations/603* 4. ***NCTM Illuminations***: Angle Sums. http://illuminations.nctm.org/Activity.aspx?id=3546

Examples of Essential Questions for the Unit	Standards for Mathematical Practice
1. How are congruence and similarity alike? How are they different? 2. What geometric attributes are considered when comparing two shapes? 3. What geometric attributes are considered when examining angles, lines, and triangles? 4. How can transformations help us understand similarity? 5. What strategies could be used to prove two figures similar? 6. What is the relationship between transformations that produce congruent figures and transformations that produce similar figures?	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Performance Objectives for Unit 2 *Students will grow in their ability to:*	**Guiding Questions for Implementing Standards for Mathematical Practices #3 and #4**
1. Understand similarity in terms of similarity transformations. 2. Describe similar figures using a sequence of transformations that preserve angle measures and have proportional sides. 3. Understand that similar figures are produced from dilations. 4. Apply the concept of similarity to write similarity statements.	1. What mathematical evidence supports your thinking? 2. What made you choose that strategy? 3. How can you be sure that …? 4. How could you prove that …? 5. Will your approach still work if …?

5. Reproduce scaled figures on grid paper. 6. Understand that proportionality is a numerical relationship that forms a straight line on the coordinate graph. 7. Prove theorems involving similarity. 8. Identify angles created when parallel lines are cut by a transversal. 9. Justify that the sum of interior angles add up to 180°. 10. Justify that the exterior angle of a triangle is equal to the sum of the two remote interior angles.	6. What were you considering when …? 7. How did you decide on that strategy? 8. How did you test whether or not your approach is correct? 9. How did you decide what the problem was asking you to find? 10. Did you initially try a method that did not work? What hunches might you have for why it didn't work? 11. What is the same and what is different about …? 12. How might you demonstrate a counterexample? 13. What mathematical model might you construct to represent the problem? 14. What are some ways to represent the quantities? 15. Where do you see one of the quantities in the task in your solution? 16. What are some ways to visually represent …? 17. What might be an expression or equation that matches the … (diagram, figure, table, etc.)? 18. Would it help to create a mathematical model (diagram, graph, table, etc.)?

Unit 3 Expressing Geometric Properties with Equations		
Concepts/Clusters	**Standards to Support CC Geometry**	**Suggested Resources**
Expressing Geometric Properties with Equations	**8.EE.5:** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example compare a distance-time graph to a distance-time equation to determine which of the two moving objects has greater speed. **8.EE.6:** Use similar triangles to explain why the slope m	1. ***MARS Task***: Modeling Car Skid Marks. http://map.mathshell.org/materials/download.php?fileid=1548 2. ***MARS Task:*** A Measure of Slope. http://map.mathshell.org/materials/download.php?fileid=1533 3. ***MARS Task:*** Real-Life Equations. http://map.mathshell.org/materials/download.php?fileid=1529 4. ***MARS Task:*** Interpreting Distance-Time Graphs. http://map.mathshell.org/materials/download.php?fileid=1521 5. ***NCTM Illuminations***: Fictional Stairs. http://illuminations.nctm.org/uploadedFiles/Content/Lessons/Resources/6-8/HillStepsSlopes-FictionalStairs.pdf

	is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equations $y = mx$ for a line through the origin and the equation $y = mx+b$ for a line intercepting the vertical axis at b.	6. ***MARS Task:*** Finding Shortest Routes: The Schoolyard Problem. http://map.mathshell.org/materials/download.php?fileid=1540 7. ***NCTM***: As the Crow Flies. http://www.nctm.org/uploadedFiles/Journals_and_Books/Books/FHSM/RSM-Task/Crow.pdf

Examples of Essential Questions for the Unit	**Standards for Mathematical Practice**
1. Why are there multiple strategies for solving a linear equations? 2. What are some reasons to find the solution of a linear equation? 3. What strategies use coordinates to prove geometric theorems algebraically? 4. How might coordinate geometry be used to prove theorems algebraically?	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Performance Objectives for Unit 3 *Students will grow in their ability to:*	**Guiding Questions for Implementing** **Standards for Mathematical Practices #5 and #6**
1. Express geometric properties with equations. 2. Interpret the unit rate of proportional relationships as the slope of the graph. 3. Graph proportional relationships. 4. Sketch and interpret graphs. 5. Determine unknown angle measures by writing and solving algebraic equations based on relationships between angles. 6. Find the slope of a line. 7. Determine the y-intercept of a line. 8. Derive equations of the form $y = mx + b$ for a line through the origin.	1. What mathematical tools could we use to visualize and represent the situation? 2. What information have we been given? 3. What do you know that is not stated explicitly in the problem? 4. What approach are you considering trying first? 5. In this situation, what might be helpful to use (a ruler, graph paper, number line, diagram, patty paper, calculator, manipulative, etc.)? 6. What can using a … show us that … may not? 7. What might it be helpful to use a …? 8. What mathematical terms apply in this situation? 9. How did you know your solution was reasonable?

9. Derive equations of the form *y* = *mx* + *b* for a line intercepting the vertical axis at the y-intercept, *b*. 10. Identify characteristics of similar triangles.	10. Explain how you might show that your solution satisfies the problem. 11. Is there a more efficient strategy? 12. What symbols or mathematical notations are important in this problem? 13. What domain-specific language can you use to explain …? 14. How might you test your solution to see if it answers the problem?

Unit 4
Similarity, Right Triangles, Trigonometry and Probability

Concepts/Clusters	Standards to Support CC Geometry	Suggested Resources
Similarity, Right Triangles and Trigonometry	**8.G.7:** Apply the Pythagorean Theorem to determine side lengths in right triangle in real-world and mathematical problems in two and three dimensions. **7.G.5:** Use facts about supplementary, complementary, vertical and adjacent angles in multi-step problems to write and solve simple equations for an unknown angle in a figure.	1. ***MARS Task:*** The Pythagorean Theorem. http://map.mathshell.org/materials/lessons.php?taskid=408&subpage=concept 2. ***NCTM Illuminations***: Corner to Corner. http://illuminations.nctm.org/Lesson.aspx?id=4082 3. ***Math Is Fun:*** Supplementary and Complementary. http://www.mathsisfun.com/geometry/supplementary-angles.html 4. ***XP Math:*** Complementary and Supplementary Angle Pairs Practice. http://www.xpmath.com/forums/arcade.php?do=play&gameid=103 5. ***Engage NY***: Introduction to Irrational Numbers Using Geometry. http://www.engageny.org/sites/default/files/resource/attachments/math-g8-m7-teacher-materials.pdf
Geometric Measurement and Dimension	**6.G.4**: Represent three-dimensional figures using nets made up of rectangles and triangles and use the nets to find the surface area of these figures applying this techniques in the context of solving real world mathematical problems **7.G.4**:	1. ***MARS Task:*** Optimizing Security Cameras. http://map.mathshell.org/materials/lessons.php?taskid=482&subpage=problem 2. ***MARS Task:*** Optimizing: Packing It In. http://map.mathshell.org/materials/download.php?fileid=1554 3. ***MARS Task:*** Using Dimensions: Designing a Sports Bag. http://map.mathshell.org/materials/download.php?fileid=1228 4. ***MARS Task:*** The Area of a Circle.

Concepts/Clusters	Standards to Support CC Geometry	Suggested Resources
	Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. **8.G.9**: Know the formulas for the volumes of cones and cylinder, and spheres and use them to solve real world and mathematical problems	http://map.mathshell.org/materials/lessons.php?taskid=597&subpage=concept 5. ***Dan Meyer:*** The Ticket Roll. www.101qs.com/66 6. ***Inside Mathematics:*** Polly Gone. http://www.insidemathematics.org/problems-of-the-month/pom-pollygone.pdf 7. ***NCTM Illuminations:*** Popcorn, Anyone? http://illuminations.nctm.org/Lesson.aspx?id=2927 8. ***NCTM Illuminations:*** Popcorn Cylinders Anyone? http://illuminations.nctm.org/uploadedFiles/Content/Lessons/Resources/6-8/Popcorn-AS-Cylinders.pdf 9. ***NCTM Illuminations:*** Cubed Cans. http://illuminations.nctm.org/uploadedFiles/Content/Lessons/Resources/6-8/CubedCans-AS.pdf 10. ***Dan Meyer:*** Popcorn Picker. http://threeacts.mrmeyer.com/popcornpicker/ 11. ***Engage NY:*** Module 5, Lesson 9: Examples of Functions from Geometry. http://www.engageny.org/sites/default/files/resource/attachments/math-g8-m5-teacher-materials.pdf
Conditional Probability and Rules of Probability	**7.SP.5**: Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the even occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely even, a probability around ½ indicates an even that is neither unlikely nor likely and a probability near 1 indicates a likely event.	1. ***MARS Task:*** Evaluating Statements about Probability. http://map.mathshell.org/materials/lessons.php?taskid=225&subpage=concept 2. ***MARS Task:*** Probability Games. http://map.mathshell.org/materials/lessons.php?taskid=596&subpage=concept 3. ***AAAS***: Marble Mania http://sciencenetlinks.com/interactives/marble/marblemania.html 4. ***NCTM Illuminations:*** Random Drawing Tool http://illuminations.nctm.org/Activity.aspx?id=3532

Concepts/Clusters	Standards to Support CC Geometry	Suggested Resources
		5. ***Engage NY:*** Statistics and Probability. http://www.engageny.org/sites/default/files/resource/attachments/math-g7-m5-teacher-materials.pdf
Using Probability to Make Decisions	**7.SP.8:** Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify he outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: if 40% of donors have type *A* blood, what is the probability that it will take at least 4 donors to find one with type *A* blood?	1. ***NCTM Illuminations:*** What Are My Chances? http://illuminations.nctm.org/Lesson.aspx?id=2895 2. **NCTM ILLUMINATIONS:** Sticks and Stones. http://illuminations.nctm.org/Lesson.aspx?id=1915 3. ***INSIDE MATHEMATICS:*** Fair Games. http://www.insidemathematics.org/problems-of-the-month/pom-fairgamescolor.pdf 4. ***INSIDE MATHEMATICS:*** Game Show. http://www.insidemathematics.org/problems-of-the-month/pom-gameshow.pdf 5. ***College Preparatory Mathematics:*** Statistics Supplement. http://www.cpm.org/pdfs/standards/stats/Statistics%20Supplement%20CCG%20TV.pdf

Examples of Essential Questions for the Unit	Standards for Mathematical Practice

1. What is the connection between the distance formula and the Pythagorean Theorem? 2. What is a strategy to determine with accuracy whether a given triangle is a right triangle? 3. What is a strategy to calculate with accuracy the distance between any two points? 4. What is the importance of 0 and 1 when examining the probability of an event? 5. What efficient strategies can be used to help determine the probability of a chance event? 6. What efficient strategies can be used to help determine the likeness of compound events to occur?	1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.

Performance Objectives for Unit 4 *Students will grow in their ability to:*	**Guiding Questions for Implementing** **Standards for Mathematical Practices #7 and #8**
1. Define trigonometric ratios and solve problems involving right triangles. 2. Build capacity in knowing common Pythagorean Triples. 3. Know that 3-D figures can be represented by nets. 4. Apply prior knowledge of finding the area of rectangles and triangles to a net and combining the areas of each shape to represent the surface area of the 3D figure. 5. Solve real world problems involving surface area and nets. 6. Grasp geometric measurement and dimensions. 7. Understand the relationship between radius and diameter. 8. Understand that the ratio of circumference to diameter can be expressed as *pi*. 9. Informally derive the relationship between circumference and area of a circle. 10. Geometric measurement and dimensions 11. Understand that knowing the formula for volume refers to the relationship between the area of the base and the height of the figure. 12. Comprehend the relationship between the volume of a cylinder	1. What observations have you made about …? 2. What do you notice when …? 3. What parts of the problem might you eliminate? 4. How would you know if … makes a pattern? 5. What useful ideas have we learned before that come in handy when solving this problem? 6. How does this relate to …? 7. In what ways might this problem connect to other mathematical concepts? 8. Will the same strategy work in other situations? 9. Is this always true, sometimes true, or never? 10. How would you prove that …? 11. What is happening in this situation? 12. Could we make a mathematical rule for …? 13. What mathematical consistencies do you notice? 14. What predictions or generalizations can this pattern support?

Performance Objectives for Unit 4 *Students will grow in their ability to:*	Guiding Questions for Implementing Standards for Mathematical Practices #7 and #8
and that of a cone. 13. Solve real-world application problems. 14. Draw conclusions to determine that a greater likelihood occurs as the number of favorable outcomes approaches the total number of outcomes. 15. Know that probability is expressed as a number between 0 and 1. 16. Create visual representations of data. 17. Develop probability models to find the probability of events. 18. Use probability to make decisions. 19. Predict frequencies of outcomes. 20. Define and describe compound events. 21. Identify the outcomes in the sample space for a relevant event 22. Choose appropriate methods (such as organized lists, tables and tree diagrams) to represent sample spaces for compound events. 23. Compare experimental probability to theoretical probability.	

INSTRUCTIONAL STRATEGIES FOR IMPLEMENTING THE STANDARDS FOR MATHEMATICAL PRACTICES

The following instructional strategies represent just a short list of strategies recommended to ensure student engagement with the Standards for Mathematical Practices, as well as the Common Core College and Career Readiness Anchor Standards for Literacy (Reading, Writing, Speaking, and Listening).

Instructional Strategies	Description
1. Activating Prior Knowledge	*Activating Prior Knowledge* allows students to connect concepts learned previously to content being taught at the time. This strategy ensures coherence throughout the course.
2. Card Sort	Students have problems or scenarios on cards and have to sort, categorize or organize the cards into specific groups.
3. Carousel	*Carousel* allows for students to collaborate on a particular problem in groups of 3-4. Students take turns with working on a problem, passing it around, round-robin style as they add to solving the problem, while checking the work of others throughout the process.

Instructional Strategies	Description
4. Corners	The *Corners* strategy works by having students select from four options (which are posted in each of the corners of the room). Once students have chosen their corners, they may be asked to defend their choice and listen to their peers' reasons for choosing a particular option, or corner.
5. Error Analysis	This strategy calls for students to analyze a problem that has been solved incorrectly. They must work with a partner to figure it out, discuss their findings, write the correct solution, and write a few paragraphs on their process and thinking.
6. Exit Tickets	*Exit Tickets* help students synthesize the learning of the day's lesson. These can be simple questions or quick-writes for students to submit at the end of the period.
7. Graphic Organizer	*Graphic Organizers* help students arrange and compartmentalize particular chunks of information. These may be in the form of fold-ables, thinking maps, charts, a Frayer-model, and so on.
8. Multiple Representations	This strategy allows students to create multiple representations in the form of pictures, tables, graphs, equations, formulas, and other models to analyze and interpret data and/or other information.
9. Poster Presentations	Students use large flip chart paper (during any stage of instruction) to collaborate, provide visuals, and solve problems. After each group is done creating posters, they may present them to the larger group or conduct a "gallery walk" to see each group's findings.
10. Questioning the Text	This strategy allows students to question the text, the problem or the situation. They can annotate and collaborate to incite discourse in the classroom.
11. Role Play	With *Role Play* students may act out or assume the role of a character or situation in order to set up (and/or solve) a problem.
12. Sentence Frames	Sentence Starters give students a structure for sharing their thoughts and ideas and guide accountable talk. Examples for use include: I think the best way to solve this problem is … I would not solve it this way because … I (agree/disagree) because … I don't think that will work because … I tried … I think … will happen. I solved the problem like this because … What if we tried … I have another approach to the problem. What about … I understand what you are saying, but what about … Now I understand … because … I agree with … because … At first, I though … but, now I think … I agree with … because … What I hear you saying is … I don't understand … but, I do understand … Another approach to this problem could be …

Instructional Strategies	Description
13. Think Alouds	In *Think Alouds*, students orally dictate their thought process to show their comprehension of a problem, text or situation. It helps when trying to make sense of problems and considering different access points to solving a problem.
14. Think-Pair-Share	Students engage in *Think-Pair-Share* by initially thinking about a problem or situation independently, then pair up with a classmate to share their thoughts or ideas.
15. Think-Ink-Pair-Share	An alternative to *Think-Pair-Share* is *Think-Ink-Pair-Share* which allows students to think independently, write their thoughts on paper, and then pair up with a partner to share their thoughts.

SAMPLE TASKS TO SUPPORT CC GEOMETRY

The following are a few examples to help support student learning in this course. These examples are not meant to be exhaustive, but are representative of the types of tasks that should be afforded to students enrolled in the course.

Sample Tasks to Support Common Core Geometry Tutorial Lab

CCSS Standard: 7.G.2: Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

Deconstructed Standard: Students make informal geometric constructions and draw geometric shapes with given parameters, including parallel lines, angles, perpendicular lines, line segments, points, etc. Students also explore whether any three side lengths will create a triangle, whether a triangle can have more than one obtuse angle, as well as other characteristics of angles that create triangles.

Task:

Keanu wants to construct a triangular frame for the front of his beach tent. But, he only has one piece of wood that happens to be 16 feet long that he must use to construct all 3 sides of this triangular frame. What are at least 3 possible dimensions that Keanu could use to construct the frame for the front of his tent? Make a recommendation for which dimension you think is best and explain your reasoning.

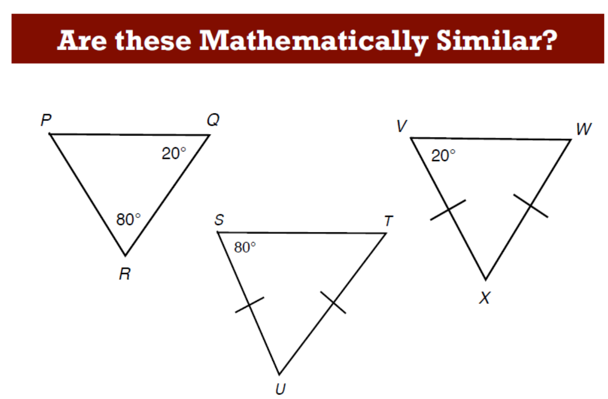
Standards for Math Practice:
MP1: Make sense of problems and persevere in solving them.
MP2: Reasons abstractly and quantitatively.
MP3: Construct viable arguments and critique the reasoning of others.
MP6: Attend to precision.

CCSS Standard: 8.G.4: Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

Deconstructed Standard: Students understand that similar figures have congruent angles and sides that are proportional. They also should understand that similar figures are produced from dilations. They should also be able to describe a sequence of similarity transformations that maps one exactly onto the other.

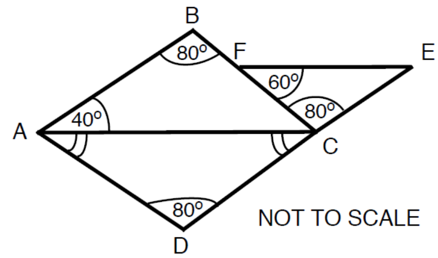
Task:

Students use mathematical reasoning to answer the question, "Are these triangles mathematically similar?" Students are encouraged to justify their reasoning carefully and use only the information on the diagrams.



Checking for Similarity

Are any of the triangles ABC, CEF and ACD mathematically similar?



For this task in its entirety, go to ***MARS Task*** website***:***
http://map.mathshell.org/materials/download.php?fileid=1372

Standards for Math Practice:
MP1: Make sense of problems and persevere in solving them.
MP2: Reasons abstractly and quantitatively.
MP3: Construct viable arguments and critique the reasoning of others.

CCSS Standard: 8.G.1: Verify experimentally the properties of rotations, reflections, and translations: a) Lines are taken to lines, and line segments to line segments of the same length; b) Angles are taken to angles of the same measure; c) Parallel lines are taken to parallel lines.

8.G.2: Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

Deconstructed Standard: Students understand that every point of the pre-image moves the same distance and direction to form the image. They understand that these transformations produce images of exactly the same size and shape, preserving distance and angle measure. They explore figures created from translations, reflections, and rotations using compasses, protractors and rulers, manipulatives, dynamic geometry software, transparencies, and/or patty paper. Students also understand that rotations, reflections, and translations are examples of rigid transformations and that two shapes are congruent if there is a sequence of rigid motions in the plane that takes one shape exactly onto the other.

CCSS Standard: 8.EE.5: Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example compare a distance-time graph to a distance-time equation to determine which of the two moving objects has greater speed.

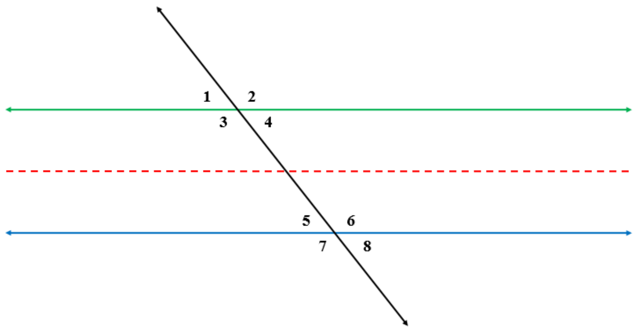
Deconstructed Standard: Students are able to graph proportional relationships. They can also compare and interpret two different proportional relationships represented in different ways via graphs, tables, and/or equations. Students are also able to identify the unit rate (or slope) in graphs, tables, and/or equations.

Task:

The diagram below shows a distance-time graph. The sloping line shows the journey of a bus from *City A* to *City B*.

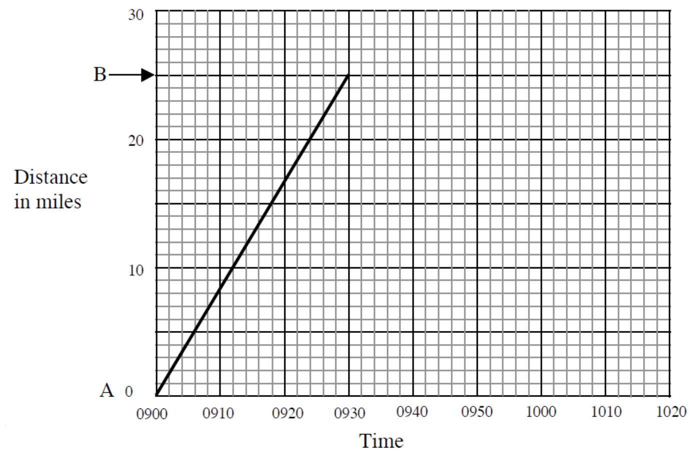
Task:

Use the diagram below to answer the following questions.



What information can be obtained if we were to cut between the two intersections on the dotted red line and slide one set directly onto the other? Identify a sequence of transformations that map ∡2 directly onto ∡3, ∡6 and ∡7.

Standards for Math Practice:
MP1: Make sense of problems and persevere in solving them.
MP2: Reasons abstractly and quantitatively.
MP3: Construct viable arguments and critique the reasoning of others.
MP5: Use appropriate tools strategically.
MP6: Attend to precision.
MP7: Look for and make use of structure.
MP8: Look for and express regularity in repeated reasoning.



1. The bus leaves *City A* at 9:00 am (0900) and arrives at *City B* at 9:30 am (0930). How far is it from *City A* and *City B*? How long does the bus journey take? How do you know?

2. Another bus leaves *City B* at 0900 and arrives at *City A* at 0930. Draw a line on the diagram to show the journey of this second bus. At about what time will the two buses pass each other? How do you know?

3. The information for a completely different bus traveling from *City* A to *City B* is given as: $y = \frac{4}{7}x$. Whereas, x is the time in minutes and y is the distance in miles. Which bus is traveling at a great speed if this bus is compared to the original bus in the diagram? Justify your answer with evidence.

Parts of this task has been modified, but originally based on a 2009 MARS Task called, *Buses*.

Standards for Math Practice:
MP1: Make sense of problems and persevere in solving them.
MP2: Reasons abstractly and quantitatively.
MP3: Construct viable arguments and critique the reasoning of others.
MP4: Model with mathematics.
MP5: Use appropriate tools strategically.
MP6: Attend to precision.
MP7: Look for and make use of structure.

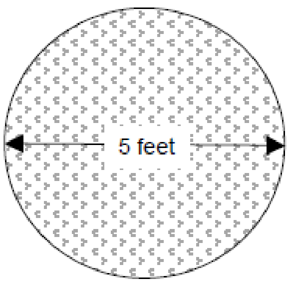
CCSS Standard: 7.G.4: Know the formulas for the area and circumference of a circle and use them to solve problems;

give an informal derivation of the relationship between the circumference and area of a circle.

Deconstructed Standard: Students understand the relationship between the circumference and the diameter of a circle and know that this ratio can be expressed as *pi*. From this relationship, students are able to informally derive the formulas for the circumference and the area of a circle and use these to solve problems.

Task:

Hank works at a factory that makes rugs. The edge of each rug is bound with braid. Hank's job is to cut the correct length of braid for each rug.



1. If the circular rug above has a diameter of 5 feet, how much braid will Hank need to go all the way around this circular rug? Give your answer in whole feet and be prepared to explain your rationale.

2. There are also plans to make a semi-circular rug which also has a diameter of 5 feet. Hank thinks that this rug will need half as much braid as the circular rug. Sketch this semi-circular rug and explain why Hank is ***not*** correct. How much braid will Hank actually need for this rug?

Parts of this task has been modified, but originally based on a 2007 MARS Task called, *Rugs*.

Standards for Math Practice:
MP1: Make sense of problems and persevere in solving them.
MP2: Reasons abstractly and quantitatively.
MP3: Construct viable arguments and critique the reasoning of others.
MP6: Attend to precision.
MP7: Look for and make use of structure.

Multi-tier Mathematics Interventions

In the practice guide, *Assisting Students Struggling with Mathematics: RtI for Elementary and Middle Schools* (Gersten, R., et. al. (2009)), the author presents evidence for the effectiveness of combinations of systematic and explicit instruction that include teacher demonstrations and think-alouds early in the lesson, unit, or module; student verbalization of how a problem was solved; scaffolded practice; and, immediate corrective feedback. In instruction that is systematic, concepts are introduced in a logical, coherent order and students have many opportunities to apply each concept. **Below are recommendations applicable to Geometry:**

Recommendation 1. Screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk.
Recommendation 3. Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.
Recommendation 4. Interventions should include instruction on solving word problems that is based on common underlying structures.
Recommendation 5. Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.
Recommendation 6. Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.
Recommendation 7. Monitor the progress of students receiving supplemental instruction and other students who are at risk.

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