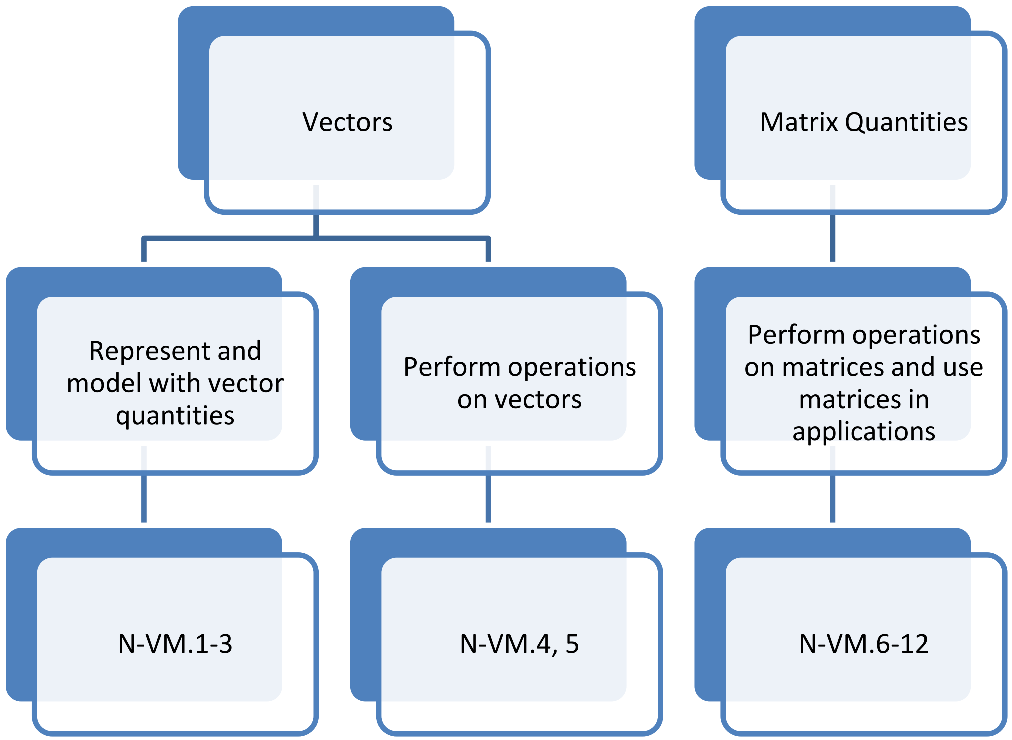
Honors Advanced Mathematics

Unit 5

Vector and Matrix Quantities



Honors Advanced Math - UNIT 5
Vectors and Matrix Quantities

Critical Area: Students work with vectors, representing them geometrically and perform operations with them. They connect the notion of vectors to the complex numbers. Students also work with matrices and their operations, experiencing for the first time an algebraic system in which multiplication is not commutative. Finally, they see the connection between matrices and transformations of the plane, namely: that a vector in the plane can be multiplied by a 2x2 matrix to produce another vector, and they work with matrices from the point of view of transformations. They also find inverse matrices and use matrices to represent and solve linear systems.

CLUSTERS	COMMON CORE STATE STANDARDS
Represent and model with vector quantities	**Number and Quantity – Vector and matrix Quantities** **N-VM.1**. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\mathbf{v}$, $\lvert\mathbf{v}\rvert$, $\lVert\mathbf{v}\rVert$, $\mathbf{v}$). **N-VM.2**. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. **N-VM.3**. (+) Solve problems involving velocity and other quantities that can be represented by vectors.
Perform operations on vectors	**A-VM.4**. (+) Add and subtract vectors a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction $\mathbf{v} - \mathbf{w}$ as $\mathbf{v} + (-\mathbf{w})$, where $-\mathbf{w}$ is the additive inverse of $\mathbf{w}$, with the same magnitude as $\mathbf{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. **A-VM.5**. (+) Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(vx, vy) = (cvx, cvy)$. b. Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $\lVert c\mathbf{v}\rVert = \lvert c\rvert\mathbf{v}$. Compute the direction of cv knowing that when $\lvert c\rvert\mathbf{v} \neq 0$, the direction of $c\mathbf{v}$ is either along $\mathbf{v}$ (for $c>0$) or against $\mathbf{v}$ (for $c<0$)
Perform operations on matrices and use matrices in applications	**A-VM.6**. (+) Use matrices to represent and manipulate data, e.g. to represent payoffs or incidence relationships in a network. **A-VM.7**. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

	A-VM.8. (+) Add, subtract, and multiply matrices of appropriate dimensions. **A-VM.9**. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. **A-VM.10**. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix and multiplicative inverse. **A-VM.11**. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformation of vectors. **A-VM.12**. (+) Work with 2 x 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area
MATHEMATICAL PRACTICES	
1. **Make sense of problems and persevere in solving them.** 2. **Reason abstractly and quantitatively.** 3. **Construct viable arguments and critique the reasoning of others.** 4. **Model with mathematics.** 5. **Use appropriate tools strategically.** 6. **Attend to precision.** 7. **Look for and make use of structure.** 8. **Look for and express regularity in repeated reasoning.**	

★**Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.**

(+) Indicates additional mathematics to prepare students for advanced courses.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
• The addition of complex numbers is connected to the addition of vectors. • Matrices could be used to represent and manipulate data, e.g. to represent payoffs or incidence relationships in a network. • Vectors and polar coordinates are useful in solving real-world problems. • Matrix operations could be performed on matrices and it can be an approach for solving systems of equations.	1) How are complex number addition connected to vector addition? 2) Why are functions and relations represented by vectors? 3) Why are functions represented by polar equations? 4) How are complex numbers connected to polar coordinates?	• horizontal/Vertical component • magnitude • modulus • vector quantity • scalar quantity • initial point • terminal point • position vector • scalar product • unit vector

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
		• equivalent vector • vector plane • resultant (sum)

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
• *Pick's Theorem as a System of Equations:* A-VM.6 The main problem in this lesson is to determine the values of the coefficients and the constant term in Pick's Theorem. In particular, what are the values of coefficients a and b, as well as the constant term c, in the following equation: *Area = a (Number of Perimeter Pins) + b (Number of Interior Pins) + c* http://illuminations.nctm.org/Lesson.aspx?id=2089 • *Sums of Vectors and Their Properties:* A-VM.4 This lessons illustrates how using a dynamic geometrical representation can help students develop an understanding of vectors and their properties, as described in the Number and Operations Standard. Students manipulate two vectors to control the movement of a plane in a game-like setting. Students extend their knowledge to further investigate the system of vectors. http://illuminations.nctm.org/Lesson.aspx?id=1590 • *Components of a Vector:* N-VM.1 In this lesson, students manipulate a velocity vector to control the movement of a car in a game setting. Students learn that vectors are composed of two	Students will investigate vectors as geometric objects in the plane that can be represented by ordered pairs, and matrices as objects that act on vectors. Through working with vectors and matrices both geometrically and quantitatively, students discover that vector addition and operations observe their own set of rules (i.e. multiplication is not commutative, it is possible that AB = AC but B≠C, it is possible that A≠0 & B≠0 but AB = 0, etc…). Students find inverse matrices by hand in 2x2 cases and use technology in other cases. Provide examples of real-world problems that can be modeled by writing equations and solved with matrices. Begin with simple equations in two variables and build up to more complex equations in three or more variables that may be solved using matrices and technology applications. *For example*: Your school's academic club is planning the end of the year party. You have determined that the cost of admission is $13.50 for non-members and $10.35 for the academic club members, and there is a limit of 40 students. You have $500 to spend. Use an inverse matrix to determine how many members and how many non-members of the academic club to invite. Have students investigate of real-world problems that can be represented and modeled with vector quantities. Students need to decide on a solution path and make use of tools (i.e. calculators, dynamic geometry software, or spreadsheets). *For instance*: Given the speed of an aircraft and its bearing (coordinates) students would find the resultant	**Illuminations** 1. Use Gauss' theorem to see if the points A(3, 6), B(2, –3) and C(6, –2) generate a cube. Then look for a pattern in the coordinates of these points. Use the pattern to generate other numbers that also the pattern always work? 2. Ask students to write a letter to an absent algebra student providing an explanation of the technique used in class, why it worked, and some of the pitfalls that must be avoided in generating this system of equations. http://illuminations.nctm.org/Lesson.aspx?id=2089 PARCC - http://www.parcconline.org/sites/parcc/files/BRHSSampleItem.pdf

components: magnitude and direction. http://illuminations.nctm.org/Lesson.aspx?id=1589 **LAUSD Adopted Textbooks** Precalculus Enhanced with Graphing Utilities, 4th Edition , Sullivan & Sullivan, Pearson/Prentice Hall (2005). Precalculus Graphical, Numerical, Algebraic, 7th edition, Demana, Waits, Foley & Kennedy, Addison Wesley, Pearson Education (2007). *Pre-Calculus with Limits: A Graphing Approach*, 5th edition, Larson, Hostetler, and Edwards, Houghton/Mifflin, Boston/New York (2008). *Precalculus with Trigonometry Concepts and Applications*, 2nd edition, Foerster, Key Curriculum (2007)	speed and direction of the aircraft by simulating the velocity of wind effects on all four nautical directions. Facilitate whole class or small group instructional conversation throughout. Instructional conversation with all students, in particular English learners will benefit from scaffolds that promote use of academic language. Mathematically Speaking is a scaffold that may be used. http://camsp.net/documents/NCTM-SpeakingArticle.pdf	

LANGUAGE GOALS

Writing:
1. Students will explain in writing how vectors as geometric objects in the plane can be represented by ordered pairs, and matrices that act on vectors.
2. Students will describe in writing an understanding of vectors and their properties.
3. Students will write equations and solve with matrices to investigate real-world problems

Example: Vector multiplication by a scalar means __________.

Speaking:
4. Students will explain (orally and in writing) the mathematical processes used in class in generating systems of equations and why it worked.

Example: The variables represent__________, and the coefficients represent ___________ because__________,…

PERFORMANCE TASKS

Pre-Calculus with Limits: A Graphing Approach, 5th edition, Larson, Hostetler, and Edwards, Houghton/Mifflin, Boston/New York, 2008.

Vectors in the Plane: Page 436 #91

Vectors and Dot Products: Page 446 #61

Linear Systems & Matrices: Page 484 #78

Operations with Matrices: Page 539 #82

Applications of Matrices & Determinants: Page 567-568 #27

Vector Tasks
https://docs.google.com/document/d/1lcRE17bVBhIZizwsHWEVOhREwu-PIOafvmc-hVoAxjA/edit

http://illuminations.nctm.org/unit.aspx?id=6081

http://illuminations.nctm.org/Activity.aspx?id=3536

http://illuminations.nctm.org/Lesson.aspx?id=1589

Matrices Tasks
http://illuminations.nctm.org/unit.aspx?id=6045

FRONT LOADING	ACCELERATION	INTERVENTION
Have students work with matrices and their operations in order for them to experience that matrix multiplication is not commutative.	Students will be able to apply the arithmetic of vectors and use the concept of vector to solve real-world problems. Students will be able to use matrix methods to solve and interpret systems of linear equations	Have students use calculators or computer software to lessen the computational burden in working with matrices. Vary amounts of time devoted to exploring problems. Stress the importance of using multiple representations in the examples by showing students mathematical modeling techniques.

References:

1. National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core State Standards (Mathematics)*. Washington D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers.
2. McCallum, W., Zimba, J., Daro, P. (2011, December 26 Draft). *Progressions for the Common Core State Standards in Mathematics*. Cathy Kessel (Ed.). Retrieved from http://ime.math.arizona.edu/progressions/#committee.

3. Engage NY. (2012). New York Common Core Mathematics Curriculum. Retrieved from http://engageny.org/sites/default/files/resource/attachments/a-story-of-ratios-a-curriculum-overview-for-grades-6-8.pdf.
4. Mathematics Assessment Resource Service, University of Nottingham. (2007 - 2012). Mathematics Assessment Project. Retrieved from http://map.mathshell.org/materials/index.php.
5. Smarter Balanced Assessment Consortium. (2012). Smarter Balanced Assessments. Retrieved from http://www.smarterbalanced.org/.
6. Partnership for Assessment of Readiness for College and Career. (2012). PARCC Assessments. Retrieved from http://www.parcconline.org/parcc-assessment.
7. California Department of Education. (2013). Draft Mathematics Framework Chapters. Retrieved from http://www.cde.ca.gov/be/cc/cd/draftmathfwchapters.asp.
8. National Council of Teachers of Mathematics (NCTM) Illuminations. (2013). Retrieved from http://illuminations.nctm.org/Weblinks.aspx.
9. The University of Arizona. (2011-12). Progressions Documents for the Common Core Math Standards. Retrieved from http://ime.math.arizona.edu/progressions.