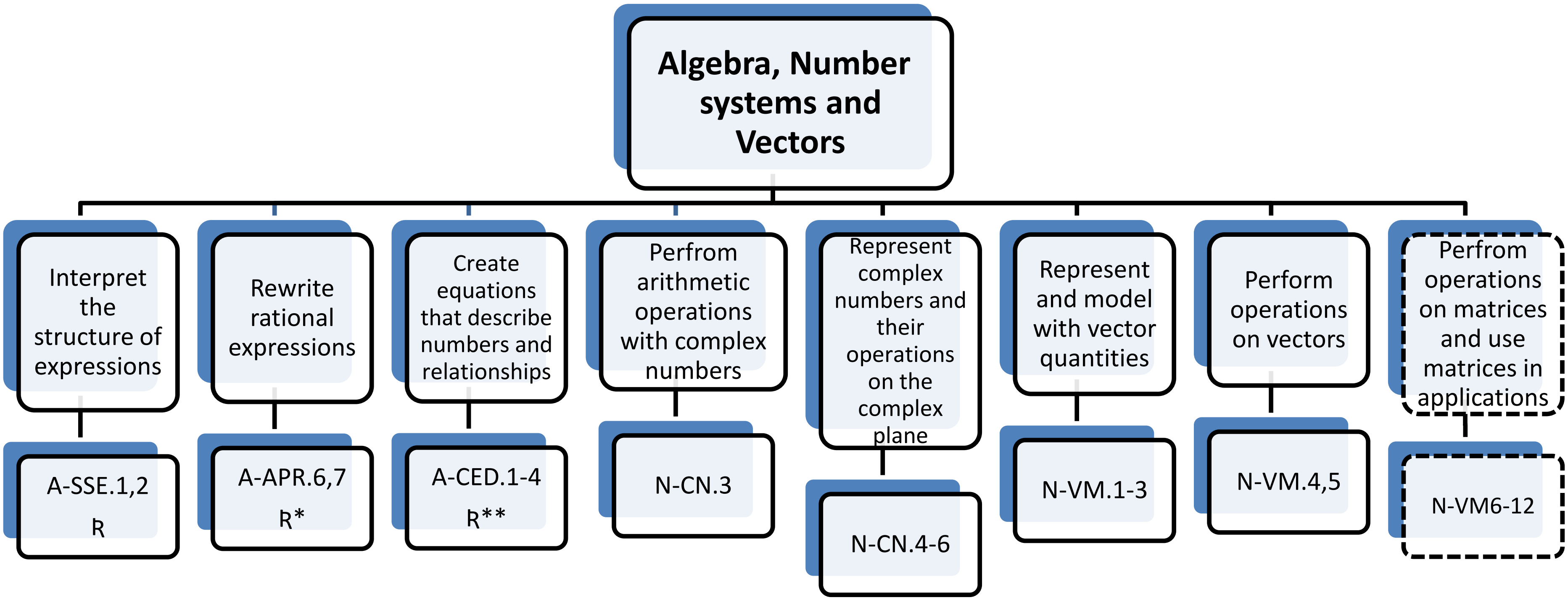
Precalculus

Unit 1

Complex Number System with Vector and Algebra



Ɽ: Review; **Ɽ***: Review and focus on more complicated examples and use computer algebra system; **Ɽ**** Review and focus on creating absolute value equations and inequalities

Precalculus – UNIT 1
Complex Number System with Vector and Algebra

Critical Area: Students extend their work in Algebra II to work with higher degree polynomials and complicated rational functions. Students see that complex numbers can be represented in the Cartesian plane and that operations with complex numbers have a geometric interpretation. They connect their understanding of trigonometry and geometry of the plane to express complex numbers in polar form. Students also work with vectors, representing them geometrically and performing operations with them. They connect the notion of vectors to the complex numbers. Students also work with matrices and their operations, experiencing for the first time an algebraic system in which multiplication is not commutative. Finally, they see the connection between matrices and transformations of the plane, namely: that a vector in the plane can be multiplied by a 2x2 matrix to produce another vector, and they work with matrices from the point of view of transformations. They also find inverse matrices and use matrices to represent and solve linear systems.

CLUSTERS	COMMON CORE STATE STANDARDS
Interpret the structure of expressions *Review*	**Algebra – Seeing Structure in Expressions** **A-SSE.1**. Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, *interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.* **A-SSE.2**. Use the structure of an expression to identity ways to rewrite it. *For example, see* $x^4 – y^4$ *as* $(x^2)^2 – (y^2)^2$*, thus recognizing it as difference of square that can be Factored as* $(x^2 – y^2)(x^2 + y^2)$
Rewrite rational expressions *[Review and focus on more complicated examples and use computer algebra system]*	**Algebra – Arithmetic with Polynomials and Rational Expressions** **A-APR.6**. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree or r(x) less that the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system **A-APR.7**. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expressions; add, subtract, multiply, and divide by rational expressions.
Create equations that describe numbers and relationships *[Review and focus on creating absolute value equations and inequalities]*	**Algebra – Creating Equations** **A-CED.1**. Create equations and inequalities in one variable **including ones with absolute value** and use them to solve problems. Include equation arising from linear functions and quadratic functions, and simple rational and exponential functions. **A-CED.2**. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

CLUSTERS	COMMON CORE STATE STANDARDS
	A-CED.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. **A-CED.4**. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.
Perform arithmetic operations with complex numbers	**Number and Quantity – Complex Number** **N-CN.3**. Find the conjugate of a complex number, use conjugates to find moduli and quotients of complex numbers.
Represent complex numbers and their operations on the complex plane	**N-CN.4**. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers); and explain why the rectangular and polar forms of a given complex number represent the same number. **N-CN.5**. Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\ i)$ *has modulus 2 and argument* 120^{o}. **N-CN.6**. Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers and its endpoints.
Represent and model with vector quantities	**Number and Quantity – Vector and matrix Quantities** **N-VM.1**. Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., **v**, |**v**|, ||**v**||, **v**). **N-VM.2**. Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. **N-VM.3**. Solve problems involving velocity and other quantities that can be represented by vectors.
Perform operations on vectors	**A-VM.4**. Add and subtract vectors a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

CLUSTERS	COMMON CORE STATE STANDARDS
	c. Understand vector subtraction **v – w** as **v + (-w**), where **-w** is the additive inverse of **w**, with the same magnitude as **w** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. **A-VM.5**. Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. b. Compute the magnitude of a scalar multiple c**v** using ‖c**v**‖ = |c|**v**. Compute the direction of cv knowing that when |c|**v** ≠ 0, the direction of c**v** is either along **v** (for c>0) or against **v** (for c<0)
Perform operations on matrices and use matrices in applications	**A-VM.6**. Use matrices to represent and manipulate data, e.g. to represent payoffs or incidence relationships in a network. **A-VM.7**. Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. **A-VM.8**. Add, subtract, and multiply matrices of appropriate dimensions. **A-VM.9**. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. **A-VM.10**. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix and multiplicative inverse. **A-VM.11**. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformation of vectors. **A-VM.12**. Work with 2 x 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.
MATHEMATICAL PRACTICES	PROGRESSION
1. Make sense of problems and persevere in solving them. **2. Reason abstractly and quantitatively.** **3. Construct viable arguments and critique the reasoning of others.** **4. Model with mathematics.**	

CLUSTERS	COMMON CORE STATE STANDARDS
5. Use appropriate tools strategically. 6. Attend to precision. **7. Look for and make use of structure.** 8. Look for and express regularity in repeated reasoning.	

★Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.

(+) Indicates additional mathematics to prepare students for advanced courses.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
• The addition of complex numbers is connected to the addition of vectors. • Matrices could be used to represent and manipulate data, e.g. to represent payoffs or incidence relationships in a network. • Vectors and polar coordinates are useful in solving real-world problems. • Complex numbers are connected to polar coordinates • Matrix operations could be performed on matrices and it can be an approach for solving systems of equations.	1) How are complex number addition connected to vector addition? 2) Why are functions and relations represented by vectors? 3) Why are functions represented by polar equations? 4) How are complex numbers connected to polar coordinates?	• arguments • Cartesian • complex plane • conjugate • horizontal/Vertical component • magnitude • modulus • polar axis • polar coordinates • polar equations • pole • position vector • real/Imaginary axis • rectangular coordinate • scalar product • unit vector

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
Illuminations • *Axonometry - Applying Complex Numbers to Art:* N-CN.4,N-CN.5 Students examine and draw representations of cubes and then learn how to analyze these representations using complex numbers. Students use what they know about operations on complex numbers to see if a drawing is an	Precalculus standards are extensions of Algebra II topics and require students to investigate and discover. Therefore, instruction would be student centered with facilitation from the teacher. Students will investigate vectors as geometric objects in the plane that can be represented by ordered pairs, and matrices as objects that act on vectors. Through working with	**Illuminations** 1. Use Gauss' theorem to see if the points A(3, 6), B(2, –3) and C(6, –2) generate a cube. Then look for a pattern in the coordinates of these points. Use the pattern to generate other numbers that also the pattern

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
accurate representation of a cube. They also learn how to generate complex numbers that will produce such representations. http://illuminations.nctm.org/Lesson.aspx?id=4228 • *Pick's Theorem as a System of Equations:*A-VM.6 The main problem in this lesson is to determine the values of the coefficients and the constant term in Pick's Theorem. In particular, what are the values of coefficients a and b, as well as the constant term c, in the following equation: *Area = a (Number of Perimeter Pins) + b (Number of Interior Pins) + c* http://illuminations.nctm.org/Lesson.aspx?id=2089 • *Sums of Vectors and Their Properties:* A-VM.4 This lessons illustrates how using a dynamic geometrical representation can help students develop an understanding of vectors and their properties, as described in the Number and Operations Standard. Students manipulate two vectors to control the movement of a plane in a game-like setting. Students extend their knowledge to further investigate the system of vectors. http://illuminations.nctm.org/Lesson.aspx?id=1590 • *Components of a Vector:* N-VM.1 In this lesson, students manipulate a velocity vector to control the movement of a car in a game setting. Students learn that vectors are composed of two components: magnitude and direction. http://illuminations.nctm.org/Lesson.aspx?id=1589	vectors and matrices both geometrically and quantitatively, students discover that vector addition and operations observe their own set of rules (i.e. multiplication is not commutative, it is possible that AB = AC but B≠C, it is possible that A≠0 & B≠0 but AB = 0, etc…). Students find inverse matrices by hand in 2x2 cases and use technology in other cases. Provide examples of real-world problems that can be modeled by writing equations and solved with matrices. Begin with simple equations in two variables and build up to more complex equations in three or more variables that may be solved using matrices and technology applications. *For example*: Your school's academic club is planning the end of the year party. You have determined that the cost of admission is $13.50 for non-members and $10.35 for the academic club members, and there is a limit of 40 students. You have $500 to spend. Use an inverse matrix to determine how many members and how many non-members of the academic club to invite. Have students investigate of real-world problems that can be represented and modeled with vector quantities. Students need to decide on a solution path and make use of tools (i.e. calculators, dynamic geometry software, or spreadsheets). *For instance*: Given the speed of an aircraft and its bearing (coordinates) students would find the resultant speed and direction of the aircraft by simulating the velocity of wind effects on all four nautical directions. Facilitate whole class or small group instructional conversation throughout. Instructional conversation with all students, in particular English learners will benefit from scaffolds that promote use of academic language. Mathematically Speaking is a scaffold that may be used. http://camsp.net/documents/NCTM-SpeakingArticle.pdf	always work? 2. Give students the following 3 points: A(–55, 148i), B(51, 94i), and C(160, 20i). Have students create a graph of their "cube" based on these three points. Does the picture seem like an accurate representation? Now have them calculate $a^2 + b^2 + c^2$. Is the answer "close" to zero? Discuss what "close to" mean in terms of complex numbers. http://illuminations.nctm.org/Lesson.aspx?id=4228 3. Ask students to write a letter to an absent algebra student providing an explanation of the technique used in class, why it worked, and some of the pitfalls that must be avoided in generating this system of equations. http://illuminations.nctm.org/Lesson.aspx?id=2089 PARCC - http://www.parcconline.org/sites/parcc/files/BRHSSampleItem.pdf

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
LAUSD Adopted Textbooks Precalculus Enhanced with Graphing Utilities, 4th Edition , Sullivan & Sullivan, Pearson/Prentice Hall (2005). Precalculus Graphical, Numerical, Algebraic, 7th edition, Demana, Waits, Foley & Kennedy, Addison Wesley, Pearson Education (2007). Pre-Calculus with Limits: A Graphing Approach, 5th edition, Larson, Hostetler, and Edwards, Houghton/Mifflin, Boston/New York (2008). Precalculus with Trigonometry Concepts and Applications, 2nd edition, Foerster, Key Curriculum (2007)		

LANGUAGE GOALS
Writing: 1. Students will explain in writing how vectors as geometric objects in the plane can be represented by ordered pairs, and matrices that act on vectors. 2. Students will describe in writing an understanding of vectors and their properties. 3. Students will write equations and solve with matrices to investigate real-world problems Example: Vector multiplication by a scalar means __________. Speaking: 4. Students will explain orally how to draw representations of cubes and how to analyze these representations using complex numbers. Listening and Speaking: 5. Students will explain (orally and in writing) the mathematical processes used in class in generating systems of equations and why it worked. Example: The variables represent__________, and the coefficients represent ___________ because__________,…
PERFORMANCE TASKS
Pre-Calculus with Limits: A Graphing Approach, 5th edition, Larson, Hostetler, and Edwards, Houghton/Mifflin, Boston/New York, 2008. **Vectors in the Plane:** Page 436 #91

Vectors and Dot Products: Page 446 #61
Trigonometric Form of a Complex Number: Page 458 #s 113 - 116
Linear Systems & Matrices: Page 484 #78
Operations with Matrices: Page 539 #82
Applications of Matrices & Determinants: Page 567-568 #27
Graphs of Polar Equations: Page 721 #72

DIFFERENTIATION 🕮		
FRONT LOADING	ACCELERATION	INTERVENTION
• Involve students to have a discussion that center around extending their knowledge of higher degree polynomials and the complex number system. • Help students see that complex numbers can be represented in the Cartesian plane and that operations with complex numbers have a geometric interpretation. • Engage students in an activity that would connect their understanding of trigonometry and geometry of the plane to expressing complex numbers in polar form. • Have students work with matrices and their operations in order for them to experience that matrix multiplication is not commutative.	• Provide examples of real-world problems that can be modeled by higher degree polynomials and rational functions. Help students connect the notion of vectors to the complex numbers. • Students will be able to apply the arithmetic of vectors and use the concept of vector to solve real-world problems. • Students will be able to use matrix methods to solve and interpret systems of linear equations.	• Have students use calculators or computer software to lessen the computational burden in working with matrices. • Vary amounts of time devoted to exploring problems. Stress the importance of using multiple representations in the examples by showing students mathematical modeling techniques.

References:

1. National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core State Standards (Mathematics)*. Washington D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers.
2. McCallum, W., Zimba, J., Daro, P. (2011, December 26 Draft). *Progressions for the Common Core State Standards in Mathematics*. Cathy Kessel (Ed.). Retrieved from http://ime.math.arizona.edu/progressions/#committee.
3. Engage NY. (2012). New York Common Core Mathematics Curriculum. Retrieved from http://www.engageny.org/resource/high-school-algebra-i.
4. Mathematics Assessment Resource Service, University of Nottingham. (2007 - 2012). Mathematics Assessment Project. Retrieved from http://map.mathshell.org/materials/index.php.
5. Smarter Balanced Assessment Consortium. (2012). Smarter Balanced Assessments. Retrieved from http://www.smarterbalanced.org/.

6. Partnership for Assessment of Readiness for College and Career. (2012). PARCC Assessments. Retrieved from http://www.parcconline.org/parcc-assessment.
7. California Department of Education. (2013). Draft Mathematics Framework Chapters. Retrieved from http://www.cde.ca.gov/be/cc/cd/draftmathfwchapters.asp.
8. National Council of Teachers of Mathematics (NCTM) Illuminations. (2013). Retrieved from http://illuminations.nctm.org/Weblinks.aspx.
9. The University of Arizona. (2011-12). Progressions Documents for the Common Core Math Standards. Retrieved from http://ime.math.arizona.edu/progressions.
10. Larson, R.; Hostetler, R.; & Edwards, B. H. (2008). *Pre-Calculus with Limits: A Graphing Approach*, 5th edition. Boston, New York: Houghton/Mifflin.
11. Sullivan, M. & Sullivan III, M. (2006). *Precalculus Enhanced with Graphing Utilities*, 4th edition. New Jersey: Pearson, Prentice Hall.
12. Demana, F.D., Waits, B.K., Foley, G.D., & Kennedy, D. (2007). *Precalculus Graphical, Numerical, Algebraic*, 7th edition. Addison Wesley, Pearson Education.
13. Foerster, P. A. (2007). *Precalculus with Trigonometry Concepts and Applications*, 2nd edition. Emeryville, CA: Key Curriculum.