Overview of the Common Core Mathematics Curriculum Map

Introduction to the Document:
Welcome to the Los Angeles Unified School District’s Common Core Mathematics Curriculum Map. The Mathematics Curriculum Map for Los Angeles Unified School District is developed as a tool for direction and clarification. It is a living document that is interactive and web-based. There are specific, precise links to provide readily accessible resources needed to appropriately meet the rigors of the common core state standards. The curriculum map is intended to be a one-stop tool for teachers, administrators, parents, and other school support personnel. It provides information on the Common Core Standards for Mathematics, assessment sample items, and suggested instructional tools organized into units providing one easy-to-read resource.

Components of the Mathematics Curriculum Map:
The curriculum map is designed around the standards for mathematics k – 12 which are divided into two sets: Practice Standards and Content standards. The Standards for Mathematical Practice are identical for each grade level. They are the expertise and understanding which the mathematics educators will seek to develop in their students. These practices are also the “processes and proficiencies” to be used as instructional “habits of mind” to be developed at all grade levels. It is critical that mathematical literacy is emphasized throughout the instructional process.

The curriculum map is grouped into four coherent units by grade level. Each unit clarifies the cluster and specific standards students are to master. In addition, the relevant Mathematical Practices and learning progressions are correlated. These sections of the curriculum map define the big idea of the unit. These four units are summarized in the Unit Organizer which provides the overview for the year.

Instructional components are specified in:
- **Enduring Understandings** are the key understandings/big ideas that the students will learn from the unit of study. These are statements that communicate the learning in a way that engages students.
- **Essential Questions** are based on enduring understandings. They are used to gain student interest in learning and are limited in number. They promote critical or abstract thinking and have the potential of more than one “right” answer. They are connected to targeted standards and are the framework and focus for the unit.
- **Standards**: Targeted (content and skills to be taught and assessed) and supporting (content that is relevant to the unit but may not be assessed; may include connections to other content areas). This includes what students have to know and be able to do (learning targets) in order to meet the standards.
Mathematical literacy is a critical part of the instructional process, which is addressed in:

- **Key Vocabulary** and **Language Goals** which clearly indicate strategies for meeting the needs of EL and SEL students

Planning tools provided are:

- **Instructional Strategies** lead to enduring understandings. They are varied and rigorous instructional strategies to teach content. They are plan experiences that reinforce and enrich the unit while connecting with the standards and assessments. Instructional strategies address individual student needs, learner perspectives, integration of technology, learning styles, and multiple intelligences.

- **Resources** and **Performance Tasks** offer concept lessons, tasks, and additional activities for learning.

- **Assessments**: This is also a listing of formative and summative Assessments to guide backwards planning. Student progress in achieving targeted standards/expected learning is evaluated. Entry-level (formative)-based on summative expectations, determine starting points for learning. Benchmark-determine progress of learning, misconceptions, strengths/weaknesses along the learning trajectory.

- **Differentiation** falls into three categories:
  - **Front Loading**: strategies to make the content more accessible to all students, including EL, SEL and students with special needs. This defines prerequisite skills needed to be successful.
  - **Enrichment**: activities to extend the content for all learners, as all learners can have their thinking advanced, and to support the needs of GATE students. These are ideas to deepen the conceptual understanding for advanced learners.
  - **Intervention**: alternative methods of teaching the standards, in which all students can have a second opportunity to connect to the learning, based on their own learning style. They guide teachers to resources appropriate for students needing additional assistance

Using the Mathematics Curriculum Map:
The guide can be thought of as a menu. It cannot be expected that one would do every lesson and activity from the instructional resources provided. To try to teach every lesson or use every activity would be like ordering everything on a menu for a single meal. It is not a logical option. Nor is it possible given the number of instructional days and the quantity of resources. That is why the document is called a "Mathematics Curriculum Map" and not a "Mathematics Pacing Plan." And, like a menu, teachers select, based on instructional data, which lessons best fit the needs of their students – sometimes students need more time with a concept and at other times, less.
An effective way to use this guide is to review and assess mathematical concepts taught in previous grades to identify potential learning gaps. From there, teachers would map out how much time they feel is needed to teach the concepts within the unit based on the data of their students' needs. For example, some classes may need more time devoted to developing expressions and equations, while another class in the same course may need more focused time on understanding the concept of functions.

The starting point for instructional planning is the standards and how they will be assessed. By first considering how the standards will be assessed, teachers can better select the instructional resources that best build mathematical understanding. There are hundreds of resources available, both publisher- and teacher-created, as well as web-based, that may be used to best teach a concept or skill. Collaborative planning, both within and among courses, is strongly encouraged in order to design effective instructional programs for students.

Learning Progressions:
The Common Core State Standards in mathematics were built on progressions: narrative documents describing the progression of a topic across a number of grade levels, informed both by research on children's cognitive development and by the logical structure of mathematics. The progressions documents can explain why standards are sequenced the way they are, point out cognitive difficulties and pedagogical solutions, and give more detail on particularly knotty areas of the mathematics. This would be useful in teacher preparation and professional development, organizing curriculum, and writing textbooks.

Standards for Mathematical Practice:
The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics (NCTM) process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).
The MIG is a living document—it is neither set in stone for all time nor is it perfect. Teachers and other users are encouraged to provide on-going feedback as to its accuracy, usability, and content. Please go to math.lausd.net and click on the 2014-2015 Curriculum Map link, and share your comments and suggestions. Your participation in making this instructional guide a meaningful and useful tool for all is needed and appreciated.

The grade level Common Core State Standards-aligned Curriculum Maps of the courses in this 2014 edition of the CCSS Mathematics Curriculum Map are the result of the collective expertise of the LAUSD Secondary Mathematics Team.

The District extends its gratitude to the following Precalculus curriculum map development team:

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Precalculus
Unit 1
Complex Number System with Vector and Algebra

Algebra, Number systems and Vectors

Interpret the structure of expressions
A-SSE.1,2
R

Rewrite rational expressions
A-APR.6,7
R*

Create equations that describe numbers and relationships
A-CED.1-4
R**

Perform arithmetic operations with complex numbers
N-CN.3

Perform and model operations on the complex plane
N-CN.4-6

Represent complex numbers and their operations on the complex plane
N-CN.4

Represent and model with vector quantities
N-VM.1-3

Perform operations on vectors
N-VM.4,5

Perform operations on matrices and use matrices in applications
N-VM6-12

R: Review; R*: Review and focus on more complicated examples and use computer algebra system; R**: Review and focus on creating absolute value equations and inequalities
Precalculus
Unit 2
Functions

Interpret functions that arise in applications in terms of the context
F-IF.4,5

Analyze functions using different representations
F-IF.7, 7d,7e

Demonstrate an understanding of functions and equations defined parametrically and graph them
F-IF.10

Graph polar coordinates and curves. Convert between polar and rectangular coordinate systems
F-IF.11

Build new functions from existing functions
F-BF.3,4
Precalculus
Unit 3
Trigonometry

Trigonometry, Geometry, and Complex Number

- Trigonometric Functions
  - Expand the domain of trigonometric functions using a unit circle (F-TF.4)
  - Model periodic phenomena with trigonometric functions (F-TF.6-7)
  - Prove and apply trigonometric identities (F-TF.9-10)

- Geometry
  - Similarity, Right Triangles & Trigonometry (G-SRT.9-11)

- Complex Numbers in Polar Form
  - Complex Numbers on the Complex Plane (N-CN.4-5)
Precalculus
Unit 4
Matrices and Conic Section

Conics, Systems & Matrices

Expressing Geometric Properties with Equations
- G-GPE.3

Solve Systems of Equations
- G-GPE.3.1
- A-REI.8, 9

Matrix Operations
- N-VM.6-12

Parametric and Polar Function
- N-CN.4-5
Critical Area: Students extend their work in Algebra II to work with higher degree polynomials and complicated rational functions. Students see that complex numbers can be represented in the Cartesian plane and that operations with complex numbers have a geometric interpretation. They connect their understanding of trigonometry and geometry of the plane to express complex numbers in polar form. Students also work with vectors, representing them geometrically and performing operations with them. They connect the notion of vectors to the complex numbers. Students also work with matrices and their operations, experiencing for the first time an algebraic system in which multiplication is not commutative. Finally, they see the connection between matrices and transformations of the plane, namely: that a vector in the plane can be multiplied by a 2x2 matrix to produce another vector, and they work with matrices from the point of view of transformations. They also find inverse matrices and use matrices to represent and solve linear systems.

<table>
<thead>
<tr>
<th>CLUSTERS</th>
<th>COMMON CORE STATE STANDARDS</th>
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<tbody>
<tr>
<td>Interpret the structure of</td>
<td><strong>Algebra – Seeing Structure in Expressions</strong></td>
</tr>
<tr>
<td>expressions Review</td>
<td>A-SSE.1. Interpret expressions that represent a quantity in terms of its context.</td>
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<tr>
<td></td>
<td>a. Interpret parts of an expression, such as terms, factors, and coefficients.</td>
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<td></td>
<td>b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)^n as the product of P and a factor not depending on P.</td>
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<tr>
<td>Rewrite rational expressions</td>
<td>A-SSE.2. Use the structure of an expression to identity ways to rewrite it.</td>
</tr>
<tr>
<td>[Review and focus on more</td>
<td>For example, see x^4 – y^4 as (x^2)^2 – (y^2)^2, thus recognizing it as difference of square that can be factored as (x^2 – y^2)(x^2 + y^2)</td>
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<tr>
<td>complicated examples and use</td>
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<td>computer algebra system]</td>
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<tr>
<td>Create equations that describe</td>
<td><strong>Algebra – Arithmetic with Polynomials and Rational Expressions</strong></td>
</tr>
<tr>
<td>numbers and relationships</td>
<td>A-APR.6. Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree or r(x) less that the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system</td>
</tr>
<tr>
<td>[Review and focus on creating</td>
<td>A-APR.7. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expressions; add, subtract, multiply, and divide by rational expressions.</td>
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<tr>
<td>absolute value equations and</td>
<td></td>
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<tr>
<td>inequalities]</td>
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<tr>
<td><strong>Algebra – Creating Equations</strong></td>
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</tr>
<tr>
<td>A-CED.1. Create equations and</td>
<td><strong>including ones with absolute value</strong> and use them to solve problems. Include equation arising from linear functions and quadratic functions, and simple rational and exponential functions.</td>
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<tr>
<td>inequalities in one variable</td>
<td></td>
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<tr>
<td>A-CED.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</td>
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<td>CLUSTERS</td>
<td>COMMON CORE STATE STANDARDS</td>
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<tr>
<td>Perform arithmetic operations with complex numbers</td>
<td><strong>A-CED.3.</strong> Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.</td>
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<td></td>
<td><strong>A-CED.4.</strong> Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.</td>
</tr>
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</table>
| Represent complex numbers and their operations on the complex plane | **Number and Quantity – Complex Number**  
**N-CN.3.** Find the conjugate of a complex number, use conjugates to find moduli and quotients of complex numbers.                                                                                                                                 |
|                                              | **N-CN.4.** Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers); and explain why the rectangular and polar forms of a given complex number represent the same number.               |
|                                              | **N-CN.5.** Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example, (-1 + √3 i) has modulus 2 and argument 120°.*                           |
|                                              | **N-CN.6.** Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers and its endpoints.                                                          |
| Represent and model with vector quantities | **Number and Quantity – Vector and matrix Quantities**  
**N-VM.1.** Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v).                                                        |
|                                              | **N-VM.2.** Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.                                                                                     |
|                                              | **N-VM.3.** Solve problems involving velocity and other quantities that can be represented by vectors.                                                                                                                      |
| Perform operations on vectors                | **A-VM.4.** Add and subtract vectors  
a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. |
|                                              | b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.                                                                                                                                 |
### CLUSTERS

Perform operations on matrices and use matrices in applications

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<tr>
<td>c. Understand vector subtraction $\mathbf{v} - \mathbf{w}$ as $\mathbf{v} + (-\mathbf{w})$, where $-\mathbf{w}$ is the additive inverse of $\mathbf{w}$, with the same magnitude as $\mathbf{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.</td>
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<tr>
<td>A-VM.5. Multiply a vector by a scalar.</td>
</tr>
<tr>
<td>a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.</td>
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<tr>
<td>b. Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $</td>
</tr>
<tr>
<td>A-VM.6. Use matrices to represent and manipulate data, e.g. to represent payoffs or incidence relationships in a network.</td>
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<tr>
<td>A-VM.7. Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.</td>
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<tr>
<td>A-VM.8. Add, subtract, and multiply matrices of appropriate dimensions.</td>
</tr>
<tr>
<td>A-VM.9. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.</td>
</tr>
<tr>
<td>A-VM.10. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix and multiplicative inverse.</td>
</tr>
<tr>
<td>A-VM.11. Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformation of vectors.</td>
</tr>
<tr>
<td>A-VM.12. Work with 2 x 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.</td>
</tr>
</tbody>
</table>

### MATHEMATICAL PRACTICES

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.

### PROGRESSION
## CLUSTERS

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<tr>
<td>5. Use appropriate tools strategically.</td>
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<tr>
<td>6. Attend to precision.</td>
</tr>
<tr>
<td>7. Look for and make use of structure.</td>
</tr>
<tr>
<td>8. Look for and express regularity in repeated reasoning.</td>
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</tbody>
</table>

*Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.

(+ Indicates additional mathematics to prepare students for advanced courses.

## ENDURING UNDERSTANDINGS

- The addition of complex numbers is connected to the addition of vectors.
- Matrices could be used to represent and manipulate data, e.g. to represent payoffs or incidence relationships in a network.
- Vectors and polar coordinates are useful in solving real-world problems.
- Complex numbers are connected to polar coordinates.
- Matrix operations could be performed on matrices and it can be an approach for solving systems of equations.

## ESSENTIAL QUESTIONS

1) How are complex number addition connected to vector addition?
2) Why are functions and relations represented by vectors?
3) Why are functions represented by polar equations?
4) How are complex numbers connected to polar coordinates?

## KEY VOCABULARY

- arguments
- Cartesian
- complex plane
- conjugate
- horizontal/Vertical component
- magnitude
- modulus
- polar axis
- polar coordinates
- polar equations
- pole
- position vector
- real/Imaginary axis
- rectangular coordinate
- scalar product
- unit vector

## RESOURCES

**Illuminations**
- Axonometry - Applying Complex Numbers to Art: N-CN.4, N-CN.5

Students examine and draw representations of cubes and then learn how to analyze these representations using complex numbers. Students use what they know about operations on complex numbers to see if a drawing is an

**INSTRUCTIONAL STRATEGIES**

Precalculus standards are extensions of Algebra II topics and require students to investigate and discover. Therefore, instruction would be student centered with facilitation from the teacher.

Students will investigate vectors as geometric objects in the plane that can be represented by ordered pairs, and matrices as objects that act on vectors. Through working with

**ASSESSMENT**

1. Use Gauss' theorem to see if the points A(3, 6), B(2, −3) and C(6, −2) generate a cube. Then look for a pattern in the coordinates of these points. Use the pattern to generate other numbers that also the pattern
accurate representation of a cube. They also learn how to generate complex numbers that will produce such representations.

http://illuminations.nctm.org/Lesson.aspx?id=4228

- **Pick’s Theorem as a System of Equations: A-VM.6**
  The main problem in this lesson is to determine the values of the coefficients and the constant term in Pick’s Theorem. In particular, what are the values of coefficients $a$ and $b$, as well as the constant term $c$, in the following equation:

$$\text{Area} = a (\text{Number of Perimeter Pins}) + b (\text{Number of Interior Pins}) + c$$

http://illuminations.nctm.org/Lesson.aspx?id=2089

- **Sums of Vectors and Their Properties: A-VM.4**
  This lesson illustrates how using a dynamic geometrical representation can help students develop an understanding of vectors and their properties, as described in the Number and Operations Standard. Students manipulate two vectors to control the movement of a plane in a game-like setting. Students extend their knowledge to further investigate the system of vectors.

http://illuminations.nctm.org/Lesson.aspx?id=1590

- **Components of a Vector: N-VM.1**
  In this lesson, students manipulate a velocity vector to control the movement of a car in a game setting. Students learn that vectors are composed of two components: magnitude and direction.

http://illuminations.nctm.org/Lesson.aspx?id=1589

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<th>RESOURCES</th>
<th>INSTRUCTIONAL STRATEGIES</th>
<th>ASSESSMENT</th>
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| Provide examples of real-world problems that can be modeled by writing equations and solved with matrices. Begin with simple equations in two variables and build up to more complex equations in three or more variables that may be solved using matrices and technology applications. For example: Your school’s academic club is planning the end of the year party. You have determined that the cost of admission is $13.50 for non-members and $10.35 for the academic club members, and there is a limit of 40 students. You have $500 to spend. Use an inverse matrix to determine how many members and how many non-members of the academic club to invite. Have students investigate of real-world problems that can be represented and modeled with vector quantities. Students need to decide on a solution path and make use of tools (i.e. calculators, dynamic geometry software, or spreadsheets). For instance: Given the speed of an aircraft and its bearing (coordinates) students would find the resultant speed and direction of the aircraft by simulating the velocity of wind effects on all four nautical directions. Facilitate whole class or small group instructional conversation throughout. Instructional conversation with all students, in particular English learners will benefit from scaffolds that promote use of academic language. Mathematically Speaking is a scaffold that may be used. http://camsp.net/documents/NCTM-SpeakingArticle.pdf | always work? 2. Give students the following 3 points: A(–55, 148i), B(51, 94i), and C(160, 20i). Have students create a graph of their "cube" based on these three points. Does the picture seem like an accurate representation? Now have them calculate $a^2 + b^2 + c^2$. Is the answer "close" to zero? Discuss what "close to" mean in terms of complex numbers.

http://illuminations.nctm.org/Lesson.aspx?id=4228 3. Ask students to write a letter to an absent algebra student providing an explanation of the technique used in class, why it worked, and some of the pitfalls that must be avoided in generating this system of equations.

RESOURCES

LAUSD Adopted Textbooks


INSTRUCTIONAL STRATEGIES

ASSESSMENT

LANGUAGE GOALS

Writing:
1. Students will explain in writing how vectors as geometric objects in the plane can be represented by ordered pairs, and matrices that act on vectors.
2. Students will describe in writing an understanding of vectors and their properties.
3. Students will write equations and solve with matrices to investigate real-world problems

Example: Vector multiplication by a scalar means __________.

Speaking:
4. Students will explain orally how to draw representations of cubes and how to analyze these representations using complex numbers.

Listening and Speaking:
5. Students will explain (orally and in writing) the mathematical processes used in class in generating systems of equations and why it worked.

Example: The variables represent __________, and the coefficients represent __________ because __________, …

PERFORMANCE TASKS


Vectors in the Plane: Page 436 #91
DIFFERENTIATION

FRONT LOADING
• Involve students to have a discussion that center around extending their knowledge of higher degree polynomials and the complex number system.
• Help students see that complex numbers can be represented in the Cartesian plane and that operations with complex numbers have a geometric interpretation.
• Engage students in an activity that would connect their understanding of trigonometry and geometry of the plane to expressing complex numbers in polar form.
• Have students work with matrices and their operations in order for them to experience that matrix multiplication is not commutative.

ACCELERATION
• Provide examples of real-world problems that can be modeled by higher degree polynomials and rational functions. Help students connect the notion of vectors to the complex numbers.
• Students will be able to apply the arithmetic of vectors and use the concept of vector to solve real-world problems.
• Students will be able to use matrix methods to solve and interpret systems of linear equations.

INTERVENTION
• Have students use calculators or computer software to lessen the computational burden in working with matrices.
• Vary amounts of time devoted to exploring problems. Stress the importance of using multiple representations in the examples by showing students mathematical modeling techniques.

References:


Precalculus – UNIT 2

Functions

Critical Area: Students develop conceptual knowledge of functions that set the stage for the learning of other standards in Precalculus. Students apply the standards in Interpreting Functions and Building Functions in the cases of polynomial functions of degree greater than two, more complicated rational functions, the reciprocal trigonometric functions, and inverse trigonometric functions. Students will examine end behavior of functions and learn how to find asymptotes. Students further their understanding of inverse functions and construct inverse functions by appropriately restricting domains. They also investigate the relationship between the graphs of sine and cosine as a function of theta and also use the parametric form of the functions where $x(\theta) = \cos(\theta)$ and $y(\theta) = \sin(\theta)$.

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<th>CLUSTERS</th>
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| Interpret functions that arise in applications in terms of the context | Functions – Interpreting Functions  
F-IF.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★  
F-IF.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble $n$ engines in a factory, then the positive integers would be an appropriate domain for the function.★  
F-IF.7d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.  
F-IF.7e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.  
F-IF.10. (+) Demonstrate an understanding of functions and equations defined parametrically and graph them. CA  
F-IF.11. (+) Graph polar coordinates and curves. Convert between polar and rectangular coordinate systems. CA |
| Building a function that models a relationship between two quantities. | Functions – Building Functions  
F-BF.3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of $k$ (both positive and negative); find the value of $k$ given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.  
|
## CLUSTERS

### COMMON CORE STATE STANDARDS

<table>
<thead>
<tr>
<th>F-BF.4</th>
<th>Find inverse functions.</th>
</tr>
</thead>
<tbody>
<tr>
<td>b. (+)</td>
<td>Verify by composition that one function is the inverse of another.</td>
</tr>
<tr>
<td>c. (+)</td>
<td>Read values of an inverse function from a graph or a table, given that the function has an inverse.</td>
</tr>
<tr>
<td>d. (+)</td>
<td>Produce an invertible function from a non-invertible function by restricting the domain.</td>
</tr>
</tbody>
</table>

### MATHEMATICAL PRACTICES

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5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

**PROGRESSION**


*Indicates a modeling standard linking mathematics to everyday life, work, and decision-making. (+) Indicates additional mathematics to prepare students for advanced courses.*

### ENDURING UNDERSTANDINGS

- Different types of relationships between quantities can be modeled with different types of functions.
- Functions and relations can be represented using polar coordinates.
- Functions and equations can be defined parametrically.
- All functions have algebraic, numerical, graphical and verbal representations.
- Operations and transformations apply to all types of functions and can be used to build new functions from existing functions.
- Graphs are visual representations of solution sets of equations and inequalities.
- The inverse functions interchange the domain and the range.

### ESSENTIAL QUESTIONS

1. What relationships exist between quantities that can be modeled by functions?
2. How can functions and relations be represented using polar coordinates?
3. Why is it important to define functions and equations parametrically?
4. What does it mean to solve equations graphically?
5. What do the domain and the range of a function represent?
6. What do the maximum and minimum represent and how do they relate to the end behavior of a function?
7. What do asymptotes represent?
8. How do we build new functions from existing functions using transformations?
9. What are the similarities and differences

### KEY VOCABULARY

- asymptotes - horizontal, vertical, oblique
- composite function
- compress/stretch
- data
- domain
- end behavior
- exponential
- functions
- increasing/decreasing
- intercepts
- inverse function
- invertible, non-invertible
- logarithmic
### ENDURING UNDERSTANDINGS
- The domain of a non-invertible function needs to be restricted in order to construct its inverse function.
- Graphs of functions can explain the observed local and global behavior of a function.
- Asymptotes represent the restricted domain or range.
- The graph of a function demonstrates the end behavior as it approaches the vertical, horizontal or oblique asymptotes.
- Real world situations can be modeled and solved by using various functions.

### ESSENTIAL QUESTIONS
- between linear, quadratic, exponential, logarithmic and polynomial functions?
- 10. How do we compare/contrast exponential and logarithmic functions?
- 11. What are inverse functions and what are they being used for?
- 12. How do we restrict the domain of a non-invertible function to produce an invertible function?

### KEY VOCABULARY
- logistic
- maximum/minimum
- modeling
- one-to-one functions
- periodicity
- polynomial
- range
- rational
- reflection over the x and y-axis
- relationship
- restricted domain
- shift
- symmetry
- transformations
- vertical/horizontal

---

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<th>RESOURCES</th>
<th>INSTRUCTIONAL STRATEGIES</th>
<th>ASSESSMENT</th>
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| **Illustrative Mathematics**
- The Canoe Trip, Variation 2 - F-IF.4
  http://www.illustrativemathematics.org/illustrations/394
- Transforming the graph of a function - F-BF.3
  http://www.illustrativemathematics.org/illustrations/742
- Building an Explicit Quadratic Function by Composition - F-BF.3
  www.illustrativemathematics.org/illustrations/744
- Graphic Representations of the Real Life Situations
  http://graphingstories.com/
- Relating the Domain of a Function to its Graph-Asymptotes and Restricted Domains

- The domains for Unit 2 are Interpreting Functions and Building Functions. Students are required to understand families of functions and the inverse of those functions. Students must be familiar with the concept and formal definition of inverse functions, namely that if 
  \[ f \circ g (x) = g \circ f (x) = x , \]
  then \( f (x) \) and \( g (x) \) are inverses of one another. Teachers should first work with evaluating functions, then composing general functions and finally composing inverse functions. Once students have mastered the composition of inverse functions, they should be made to derive the inverse functions and prove that they have found the inverse by using the above definition.

- Students should recall parent functions \( f (x) \) and then explore the effect of \( f (x) + k \), \( f (x+k) \), \( kf(x) \), \( f (kx) \) on the graph for all \( k \). The mathematical

- SBAC – http://www.smarterbalanced.org/
- PARCC -

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LAUSD Secondary Mathematics
### RESOURCES

<table>
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### INSTRUCTIONAL STRATEGIES

progressions demand that students are fluent with the
parent functions and can use them quickly to determine the
graph of transformed functions.

Students will explore the relationship between functions
and their inverses on the same coordinate plane. They will
use that understanding to then explain the connection
between the line of symmetry of the two functions and the
algebraic method of letting \( f(x) = x \) and \( x = f^{-1}(x) \)
to solve for the inverse function \( f^{-1}(x) \). Students should
then come to understand why a function needs to be one-to-
one in order to have an inverse and then why it is necessary
and possible to restrict a domain on a function to create an
invertible function.

Provide visual examples of transformed functions while
manipulating different constants in the function parameters.

An instructional conversation with all students, in
particular English learners will benefit from scaffolds that
promote use of academic language. Mathematically
Speaking is a scaffold that may be used.

http://camsn.net/documents/NCTM-SpeakingArticle.pdf

### ASSESSMENT

### LANGUAGE GOALS

**Writing:**

1) Students will explain and justify in writing the behavior of the function as it approaches horizontal and vertical asymptotes.

   *Example: As the function approaches positive infinity along the x-axis, the graph of the function approaches the horizontal asymptote from above.*

2) Students will explain (in writing and orally) the effects of transformations on a function and test that understanding for all parent functions.

   *Example: The transformation \( f(x + a) + b \), moves the parent function \(-a \) units in the horizontal direction and \( b \) units in the vertical direction.*

### LANGUAGE GOALS

3) Students will compare and contrast (in writing and orally) the differences and similarities between linear, polynomial, and exponential functions.

   *Example: All three functions increase as \( x \) increases. Polynomial and exponential functions are curves and the linear function is a line. Exponential functions will increase at a faster rate than polynomial functions.*

4) Students will write about the relationship between the inverse of functions and the concept of rotating the axes about the line of symmetry to determine the
inverse function.

*Example:* The inverse function can be determined by rotating the function of the graph about the line of symmetry. This is algebraically equivalent to interchanging the x and y values in a function and solving for y.

5) Students will write about how functions can be used in real life to facilitate repeated algorithms.

*Example:* Computers often make use of functions to run programs i.e. clicking on the icon for Internet Explorer will run a function to launch a program that connects the modem to the internet and opens a screen to a preselected page.

**Listening and Speaking:**

1) Students will participate in class discussions using specific vocabulary related to transformations and functions.

2) Students will explain and justify (orally) how to graph a function to a partner as well as restating and summarizing their partner’s explanation.

*Example:* First I __________ because ____________, second I ___________ because ____________,...

**Reading:**

1) Students will identify the relevant information and details in a passage and create a single function that represents a composition out of many subparts.

**PERFORMANCE TASKS**


**F-IF.4.**

- Enclosing the Most Area with a Fence, Page 166, 79
- Minimizing Marginal Cost, Page 144, # 87
- Norman Windows, Page 167, # 86

**F-IF.7d**

- Population Model, Page 197, # 54
- Cost of a Can, Page 210, # 61

**F-BF.4**

- Discussion and Writing, Page 270, # 84-90


**F-IF.4.**

- Modeling the Height of a Bouncing Ball, Chapter 2 Project, page 273
- Designing a Swimming Pool, Page 255, # 38

**F-IF.7d**

- Designing a Cardboard Box, Page 265, # 59

**PERFORMANCE TASKS**

- Industrial Design, Page 272, # 94 and 95
- Designing a Juice Can, Page 265, # 61

**Illustrative Mathematics**

Transforming the graph of a function - F-BF.3: [http://www.illustrativemathematics.org/illustrations/742](http://www.illustrativemathematics.org/illustrations/742)
### FRONT LOADING
- Have students recall how to graph by hand linear, quadratic and cubic functions from a table of values and then understand how to graph all parent functions.
- Involve students in a simple discussion of what an inverse means and how differs from opposite or reciprocal.
- Involve students in the processes required to solve equations and start to discuss the concept of inverse functions.
- Engage students in an activity that would involve comparing linear functions with quadratics functions, and then quadratics functions and exponential functions.
- Have students match linear, quadratic, and exponential functions with their graphs, tables, and equations.
- Get the students to explain how to solve quadratic equations by completing the square.

### ACCELERATION
- Students work in small groups with a curriculum that is conceptually demanding as well as rigorous due to the speed at which the course moves and the concepts covered. Students collaborate and concentrate on tasks for extended periods of time, to contribute to discussions, to predict and test their predictions.
- The assessments for advanced students will demand the ability to apply learned concepts to solving abstract or real world problems or summarize the patterns/concepts learned. Students will use the “Socratic Method” for posing questions to discover connections, patterns and structure.
- Students learn about the modeling of real world data with polynomial functions, rational functions, exponential functions, radical functions, logarithmic functions, and sinusoidal functions. They explore in depth the various characteristics of functions, i.e. rates of change, concavity, inverses, continuity, discontinuity and asymptotes. Students further explore functions in terms of composite and inverse functions, their transformations and periodicity.
- Students work on projects to apply these concepts to real-world problems by creating equations and exploring the graphs of those equations using technology application to determine which parts of the graph are relevant to the problem context. A modeling problem involving the most efficient way to solve an investment dilemma for stock broker will be posed to the students. They model the problem by finding a solution pathway that would optimize the profit of the brokerage firm when they invest in different stock options. Given the stock pricing which would involve volume-based discount, students will model and graph the revenue, cost, and profit functions. They would interpret the vertices, intercepts, and intersection points as well as solve.

### INTERVENTION
- Demonstrate for students how to create tables of values and to use those values to generate the graph of the function.
- Allow students to use technology to quickly generate a table of values after they have shown some skill in evaluating expressions by hand.
- Students use graphic organizers to graph functions by hand and analyze the graphs in terms of increasing, decreasing, positive, or negative; relative maximums/minimums, the symmetry and continuity.
- Using technology, students work in small groups to graph different function and compare/contrast the graphs and make conclusions.
- Students graph transformations of quadratic and cubic functions graphs and analyze the differences and similarities.
systems of equations to discover the exact number of customers that would help them maximize revenue and profit.

References:
Pre-Calculus – UNIT 3
Trigonometry

**Critical Area:** Students expand their understanding of the trigonometric functions by connecting properties of the functions to the unit circle, e.g., understanding that since that traveling $2\pi$ radians around the unit circle returns one to the same point on the circle, this must be reflected in the graphs of sine and cosine. Students extend their knowledge of finding inverses to doing so for trigonometric functions, and use them in a wide range of application problems. Students derive the addition and subtraction formulas for sine, cosine and tangent, as well as the half angle and double angle identities for sine and cosine, and make connections between among these. The relationships of general triangles using appropriate auxiliary lines result in the Laws of Sines and Cosines in general cases, and they connect the relationships described to the geometry of vectors. Students investigate the geometry of the complex numbers more fully and connect it to operations with complex numbers. In addition, students develop the notion of a vector and connect operations with vectors and matrices to transformations of the plane.

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<tr>
<th>CLUSTERS</th>
<th>COMMON CORE STATE STANDARDS</th>
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<tr>
<td>Expand the domain of trigonometric functions using a unit circle.</td>
<td><strong>Functions – Trigonometric Functions</strong></td>
</tr>
<tr>
<td>Model periodic phenomena with trigonometric functions</td>
<td><strong>F-TF.4.</strong> Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.</td>
</tr>
<tr>
<td>Prove and apply trigonometric identities</td>
<td><strong>F-TF.6.</strong> Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.</td>
</tr>
<tr>
<td>Similarity, Right Triangles &amp; Trigonometry</td>
<td><strong>F-TF.7.</strong> Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.</td>
</tr>
<tr>
<td></td>
<td><strong>F-TF.9.</strong> Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.</td>
</tr>
<tr>
<td></td>
<td><strong>F-TF.10.</strong> Prove the half angle and double angle identities for sine and cosine and use them to solve problems.</td>
</tr>
<tr>
<td></td>
<td><strong>Geometry – Similarity, Right Triangles, and Trigonometry</strong></td>
</tr>
<tr>
<td></td>
<td><strong>G-SRT.9.</strong> Derive the formula $A = \frac{1}{2}ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.</td>
</tr>
<tr>
<td></td>
<td><strong>G-SRT.10.</strong> (+) Prove the Laws of Sines and Cosines and use them to solve problems.</td>
</tr>
<tr>
<td></td>
<td><strong>G-SRT.11.</strong> (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown</td>
</tr>
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### CLUSTERS

<table>
<thead>
<tr>
<th>Complex Numbers on the Complex Plane [Revisit]</th>
<th>COMMON CORE STATE STANDARDS</th>
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#### MATHEMATICAL PRACTICES

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

#### ENDURING UNDERSTANDINGS

- Trigonometric relationships and functions could be used to model real-world phenomenon.
- Indirect measurements of lengths and angles can be used to solve a variety of problems.
- The characteristics of circular functions and their representations are useful in solving real-world problems.
- The relationship between the graph of a complex number and their operations and the conjugation of complex numbers on the complex plane can be understood.

#### ESSENTIAL QUESTIONS

1. How can the graphs of the sine, cosine, tangent functions and their inverses be compared?
2. How can you use the addition and subtraction formulas for sine, cosine, and tangent to solve problems?
3. How can you find the inverse of a trigonometric function?
4. How can you solve trigonometric equations using inverse functions?
5. How can technology be used to evaluate solutions of

#### KEY VOCABULARY

- amplitude
- asymptote
- complex number
- cosecant (csc)
- cosine (cos)
- cotangent (cot)
- coterminal angles
- even function
- inverse
- midline

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*Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.

(+ Indicates additional mathematics to prepare students for advanced courses.
### ENDURING UNDERSTANDINGS
- The proof of addition and subtraction of identities are derived from the unit circle.
- Domain must be limited to finding the inverse of a trigonometric function. Inverse functions must be used to find solutions in some modeling problems.

### ESSENTIAL QUESTIONS
- trigonometric functions?
  6) How can you graph a complex number in rectangular and polar form?
  7) What is the relationship between rectangular and polar form of a complex number?
  8) What is the importance of knowing the conjugate of a complex number?
  In terms of their respective equations, what is the difference between a circle and an ellipse?

### KEY VOCABULARY
- odd function
- period
- periodic functions
- phase shift
- polar form
- quadrant angles
- rectangular form
- secant (sec)
- sine (sin)
- tangent (tan)

### RESOURCES
**Illustrative Mathematics**
- Axonometry: N-CN.4, N-CN.5
- Graphs from the Unit Circle: F-TF.4
- Cutting Conics: G-GPE.3
- Shrinking Candles: T-TF.7
- Human Conics – G-GPE.3
  [http://illuminations.nctm.org/Lesson.aspx?id=3003](http://illuminations.nctm.org/Lesson.aspx?id=3003)

### INSTRUCTIONAL STRATEGIES
- Use the values on a unit circle to generate the graphs of the sine and cosine functions on the coordinate plane.
- Explore different ways to prove the Law of Sines and Cosines. Derive the Law of Sines from the formula of the area of the non-right triangle.
- Use properties of difference of two squares to find the modulus. Relate the modulus visually using vectors.
- Graph complex numbers and identify the magnitude of the complex number, the distance of the complex number from the origin, and the direction of the complex number from the origin.
- Express complex numbers in polar coordinate form and in rectangular form.
- Tie measures in special right triangles to values on the unit circle and use those values to generate a relationship between the angles and the corresponding locations on the unit circle.
- The algebraic proofs for sum and difference formulas for sine and cosine flow nicely once you know the cosine formulas. First use the distance formula and Pythagorean

### ASSESSMENT
**Illustrative Mathematics**
- Properties of Trigonometric Functions:
  [http://www.illustrativemathematics.org/illustrations/1704](http://www.illustrativemathematics.org/illustrations/1704)
### RESOURCES


### INSTRUCTIONAL STRATEGIES

- Use the identity to derive the cosine formulas. Then allow students to derive the formulas for sine and tangent.

- Have students explore the conic sections and describe how to cut a cone to create various conic sections.

- Import images of circles from fields from Google Earth into a coordinate grid system and find their equations.

### LAUSD Adopted Textbooks


### ASSESSMENT

### LANGUAGE GOALS

**Writing:**

1. Students will explain in writing how to prove and apply the Laws of Sines and Cosines using technical vocabulary in complex sentences.

   *Example:* To derive the Law of Sines from the formula of the area of the non-right triangle, I (draw the altitude) h from the (vertex) A of the triangle from the definition of the (sine function).

2. Students will explain (in writing and orally) the terms and definitions of the trigonometric functions; conic sections; and complex numbers.

   *Example:* To find the (amplitude) of the function, I can first find the (midline) and then find the distance to the (maximum or minimum) of the graph.

3. Students will compare and contrast in writing the differences between a circle and an ellipse.
LANGUAGE GOALS

Example: I can derive the formula $A=\frac{1}{2}ab\sin(C)$ for the area of a triangle by drawing an (auxiliary line) from a (vertex) that is (perpendicular) to the opposite side.

Listening and Speaking:
4. Students will generate class discussions using academic vocabulary related to the rectangular and polar forms of complex numbers.

Example: Complex number can be expressed in (polar coordinate) form and in (rectangular form) by ______________.

Reading:
5. Students will read a word problem and identify the language needed to create an algebraic representation in order to solve the problem.

PERFORMANCE TASKS


- F-TF.4 - Electrical Circuits, #73, page 275;
- F-TF.7 – Photography, page 329 #83;
- F-TF.9 – Standing Waves, page 385, #79; Harmonic Motion, page 386, #80,
- F-TF.10 – Railroad Track, page 397, #129; Mach Number, page 398, #128.
- G-GPE.3 – Architecture, page 678, #47-49;
- G-SRT.10 – Surveying, page 422, #38; Landau Building, page 422, #45.

DIFFERENTIATION

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<th>ACCELERATION</th>
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<tr>
<td>• Involve students to have a discussion that center around extending their knowledge of creating and analyzing systems of linear equations and inequalities. Have them use their prior knowledge of graphing linear equations to approach system of linear and quadratic equations with two variables.</td>
<td>• Consider using Application Problems found in the textbook for real-world examples that can be solved by writing an equation, and have students explore the graphs of equations using technology application.</td>
<td>• Show students how to create numerical equations and then introduce linear equations in one variable. Students can make comparisons using the numerical and linear equations.</td>
</tr>
<tr>
<td>• Engage students in an activity that would involve comparing linear equations with quadratics equations, and then quadratics equations and exponential equations.</td>
<td>• Provide examples of real-world problems that can be modeled by writing linear, polynomial, rational, absolute value, exponential, and logarithmic functions. Have students use technology to graph the functions, make tables of values, or find successive approximations resulting from the function. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.</td>
<td>• For graphing, have students make a T-chart of the equations, graph them and them analyze, find the intersection of the equations, and then explain what that means. Include a case where they would compare simple linear and quadratics equations, e.g. $y=2x$ and $y=x^2$</td>
</tr>
<tr>
<td>• Have students match linear, quadratic, and exponential functions with their graphs, tables, and equations.</td>
<td></td>
<td>• Precalculus intervention should include strategies such as targeted grouping peer and counseling grouping.</td>
</tr>
<tr>
<td>• Direct students to connect the idea of functions with trigonometry and see sine,</td>
<td></td>
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</table>
DIFFERENTIATION

FRONT LOADING
- cosine, and tangent values as functions of angle values input in radians.
- Review the definition of circles as a set of points whose distance from a fixed point is constant.
- Review the algebraic method of completing the square.
- Illustrate conic sections geometrically as cross sections of a cone.
- Have students define conic sections and illustrate it pictorially.
- If the imaginary unit $i$ is misinterpreted as -1 instead of $\sqrt{-1}$, re-establish a definition of $i$.

ACCELERATION
- Give students examples of real-world problems that can be solved by writing an equation, and have students explore the graphs of the equations using technology application to determine which parts of the graph are relevant to the problem context.
- Have students write a system of two equations in two variables where one equation is quadratic and the other is linear such that the system has no solution. Explain, using graphs, algebra and/or words, why the system has no solution.

INTERVENTION
- Use informal techniques frequently during regular class time to gauge student understanding.
- Use questioning that focuses on student thinking and reasoning to help you monitor your students.
- Incorporate writing activities and group work to observe student thinking and identify misconceptions and gaps in understanding.
- Many students who need intervention struggle to learn concepts because they may not be able to grasp abstract concepts. Whenever possible, vary your instructional techniques to include use of models, manipulatives, and technology.

References:
**Critical Area:** Students derive the equations of ellipses and hyperbolas given foci. Given a quadratic equation of the form $ax^2 + by^2 + cx + dy + e = 0$, they use the method of completing the square to put the equation in standard form; identify whether the graph of the equation is a circle, parabola, ellipse, or hyperbola as well as graph the equation. Students model situations, involving payoffs in games, economic, or geometric situations to systems of linear equations and connect the newfound knowledge of matrices to solving problems. Students investigate vectors as geometric objects in the plane that can be represented by ordered pairs, and matrices as objects that act on vectors. Students discover that vector addition and subtraction behave according to certain properties, while matrices and matrix operations observe their own set of rules. Students discover with matrices a new set of mathematical objects and operations among them that has a multiplication that is not commutative. Students expand the skills involved in working with equations into several areas: trigonometric functions, by setting up and solving equations such as $\sin 2\Theta = \frac{1}{2}$; parametric functions by making sense of the equations $x = 2t$, $y = 3t + 1$, $0 \leq t \leq 10$.

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<tr>
<th>CLUSTERS</th>
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| Translate between the geometric and the equation for a conic section | **Geometry – Expressing Geometry Properties with Equations**  
G-GPE.3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is consistent.  
G-GPE.3.1. Given a quadratic equation of the form $ax^2 + by^2 + cx + dy + e = 0$, use the method of completing the square to put the equation in standard form; identify whether the graph of the equation is a circle, parabola, ellipse, or hyperbola, and graph the equation. |
| Solve systems of equations                    | **Algebra – Reasoning with Equations and Inequalities**  
A-REI.8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.  
A-REI.9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equation (using technology for matrices of dimension 3 x 3 or greater). |
| Represent complex numbers and their operations on the complex plane | **Number and Quantity – Complex Number**  
N-CN.4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.  
N-CN.5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3}i)^3 = 8$ because $(-1 + \sqrt{3}i)$ has modulus 2 and argument 120°. |
Perform operations on matrices and use matrices in applications

**Number and Quantity – Vector and Matrix Quantities**

**N-VM.6.** (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.

**N-VM.7.** (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.

**A-VM.8.** Add, subtract, and multiply matrices of appropriate dimensions.

**A-VM.9.** Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.

**A-VM.10.** Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix and multiplicative inverse.

**A-VM.12.** Work with 2 x 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

**MATHEMATICAL PRACTICES**

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the arguments of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

**PROGRESSION**

*Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.

(+ Indicates additional mathematics to prepare students for advanced courses.
### ENDURING UNDERSTANDINGS

- The sum or difference of the distances of the foci from the directrix is consistent.
- Graphs of quadratic equations of the form $ax^2 + by^2 + cx + dy + e = 0$ can be circles, parabolas, ellipses, or hyperbolas.
- The inverse of a matrix may or may not exist.
- Matrices could be used to solve real-world problems involving systems of linear equations.
- Linear equations can be represented as a single matrix.
- The equations of ellipses and hyperbolas can be derived from the foci.

### ESSENTIAL QUESTIONS

1. What are the geometric characteristics of conics?
2. How do you identify the graphs of quadratic equations of the form $ax^2 + by^2 + cx + dy + e = 0$?
3. How do you find the inverse of a matrix?
4. How can data be represented as a matrix?
5. How can the equations of ellipses and hyperbolas derived from the foci?
6. How would you use matrices to solve system of equations?

### KEY VOCABULARY

- circle
- Cramer’s Rule
- determinant
- directrix
- eccentricity
- ellipses
- foci
- hyperbolas
- identity matrix
- inverse matrix
- Law of Sine/Cosine
- matrix
- parabola
- parametric function
- row/column
- scalar
- vector

### RESOURCES

**NCTM Illuminations**

- **Cutting Conics**: G-GPE.3
  Students explore and discover conic sections by cutting a cone with a plane. Circles, ellipses, parabolas, and hyperbolas are examined using the Conic Section Explorer tool. Physical manipulatives such as dough can optionally be used as well.  

- **Human Conics**: G-GPE.3
  Students use sidewalk chalk and rope to illustrate the locus definitions of ellipses and parabolas. Kinesthetics, teamwork, and problem solving are stressed as students take on the role of focus, directrix, and point on the conic, and figure out how to construct the shape.  
  [http://illuminations.nctm.org/Lesson.aspx?id=3003](http://illuminations.nctm.org/Lesson.aspx?id=3003)

- **Mars Orbit**: F-IF.10

**INSTRUCTIONAL STRATEGIES**

Students will explore the conic sections and describe how to cut a cone to create the various conic sections. Separate the class into 6 groups (or a multiple of 6 if your class is large). Assign two conic sections to each group. There are 6 different ways to do this: circle/ellipse, circle/hyperbola, circle/parabola, ellipse/hyperbola, ellipse/parabola, and hyperbola/parabola. Each group should create a poster summarizing what they’ve learned about their two conic sections and comparing and contrasting them.

Students will write a summary of either the ellipse or parabola construction for the benefit of a classmate who has missed the lesson. The summary should include the definition and an explanation of how the drawing technique applies the definition. Afterwards, students can exchange and critique their summary with other students.

Given parametric equations, group students and ask them to find the polar equation that will give the same shape as the

**ASSESSMENT**

1. Ask students to describe how they discovered how to cut their cones to create each conic section – circles, ellipses, parabolas, hyperbolas.
2. Give students a picture of an ellipse and a parabola with possible foci or directrix indicated. Ask them to use a ruler and right angle measure to determine and explain whether or not the figure is actually the named conic.
3. Using data regarding the distance from the Sun and the orbital periods of other planets, ask students to generate parametric equations for the orbits of the other planets in the solar system relative to the Earth.
Students will generate parametric equations to describe the position of planets relative to the Sun; then, they will combine the equations to describe the position of Mars relative to Earth.

http://illuminations.nctm.org/Lesson.aspx?id=3980

- Caesar Cipher: A-VM.10
  In this lesson, students will investigate the Caesar substitution cipher. Text will be encoded and decoded using inverse operations.
  http://illuminations.nctm.org/Lesson.aspx?id=1926

- Pick’s Theorem as a System of Equations: N-VM.6
  Students will gather three examples from a Geoboard or other representation to generate a system of equations. The solution will provide the coefficients for Pick’s Theorem.
  http://illuminations.nctm.org/Lesson.aspx?id=2089

LAUSD Adopted Textbooks


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one obtained with given parametric equations. Afterwards, students will share their explanations in a whole class discussion.

Guide students to transform a system of linear equations in two variables into matrix. Then help students to solve the resulting matrix by Cramer’s rule. For example:

\[
\begin{align*}
2x + 3y &= 6 \\
4x + 5y &= 1
\end{align*}
\]

and then solved using determinants.

Design an instruction that would help students to discover with matrices a new set of mathematical objects and operations among them that has a multiplication that is not commutative.

Engage students in an activity to investigate vectors as geometric objects in the plane that can be represented by ordered pairs, and matrices as objects that act on vectors.

Have them discover that vector addition and subtraction behave according to certain properties, while matrices and matrix operations observe their own set of rules.
**LANGUAGE GOALS**

**Writing:**
1) Students will explain and justify the process of completing the square to identify whether the quadratic equation of the form \(ax^2 + by^2 + cx + dy + e = 0\) is an ellipse, circle, parabola, or a hyperbola.
   
   *Example: I completed the process of completing the square by _____ and found that _____. This means that graph of the quadratic equation is a _______*.

2) Students will compare and contrast the differences and similarities between ellipses, circles, parabolas, and hyperbolas.
   
   *Example: If the eccentricity of a conic section is ______, then the graph is a _______*.

**Listening and Speaking:**
1) Students will generate class discussions using specific vocabulary related to solving systems of equations and operations with matrices.
   
   *Example: To solve systems of equations by (matrix), I can use the (Cramer rule) and I transform the equation in matrix and then find the determinant."

2) Students will explain and justify how to solve systems of equations using operations with matrices to a partner as well as restating and summarizing their partner’s explanation.
   
   *Example: First I __________ because ____________, second I ___________ because__________,…*

**Reading:**
1) Students will identify the relevant information and details in a passage that help them to use matrices to represent and manipulate data.

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**PERFORMANCE TASKS**


**Publisher:** Houghton Mifflin Company

**Authors:** Larson, R., Hostetler, R.

**Topic:** Matrices and Systems of Equations
- Healthcare (Page 601, Problem 70)
- Data Analysis: License Drivers (Page 610, Problem 72)
- Data Analysis: Supreme Court (Page 630, Problem 58)

**Topic:** Vectors
- Navigation (Page 459, Problem 84)
- Braking Load (Page 468, Problem 67)

**Topic:** Conics
- Suspension Bridge (Page 742, Problem 62)
- Loran (Page 761, Problem 42)
- Satellite Tracking (Page 798, Problem 58)


- Earthquake: Page 667 #35
- Road Design: Page 669 #94
- Architecture: Page 678 #49
- Navigation: Page 688 #46
- Planetary Motion: Page 727 #55

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<td>Introduce students to ellipses and help them understand that conics are like circle and parabolas.</td>
<td>Provide examples of real-world problems that can be modeled by circles, parabolas, and ellipses.</td>
<td>Students will write and graph equations in polar form.</td>
<td>Have students use calculators or computer software to lessen the computational burden in simplifying and graphing conics.</td>
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<td>Introduce students to the equations and graphs of conics and help them see the relationship between equation and graph.</td>
<td>Students will classify conics from their general equation.</td>
<td>Students will use properties of parabolas, ellipses, and hyperbolas to model and solve real-life problems.</td>
<td>Use hands-on activities to allow students to explore how conics may vary (i.e. Using a string and two thumbtacks, have students explore how to obtain ellipses that are long or narrow)</td>
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<tr>
<td>Engage students in an activity that would connect their understanding of conics to the real-world.</td>
<td>Students will use properties of parabolas, ellipses, and hyperbolas to model and solve real-life problems.</td>
<td>Through guided discovery, have students discover that vector addition and subtraction behave according to certain properties, while matrices and matrix operations observe their own set of rules.</td>
<td>Have students use technology to perform matrix operations; including addition, subtraction, and multiplication of matrices.</td>
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<td>Design a frontloading activity that would introduce students to the idea of vector addition and subtraction and matrix operations.</td>
<td>Have students resolve vectors involving forces. For example: Two students are moving a box up a ramp that is inclined 40°. One pushes on the box with a force of 40 N. The other student pulls the box with a force of 38 N at an angle of 40° from horizontal. What is the net force (magnitude and direction) on the box – that is, calculate the resultant force.</td>
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![Diagram of vector addition](image)
References: