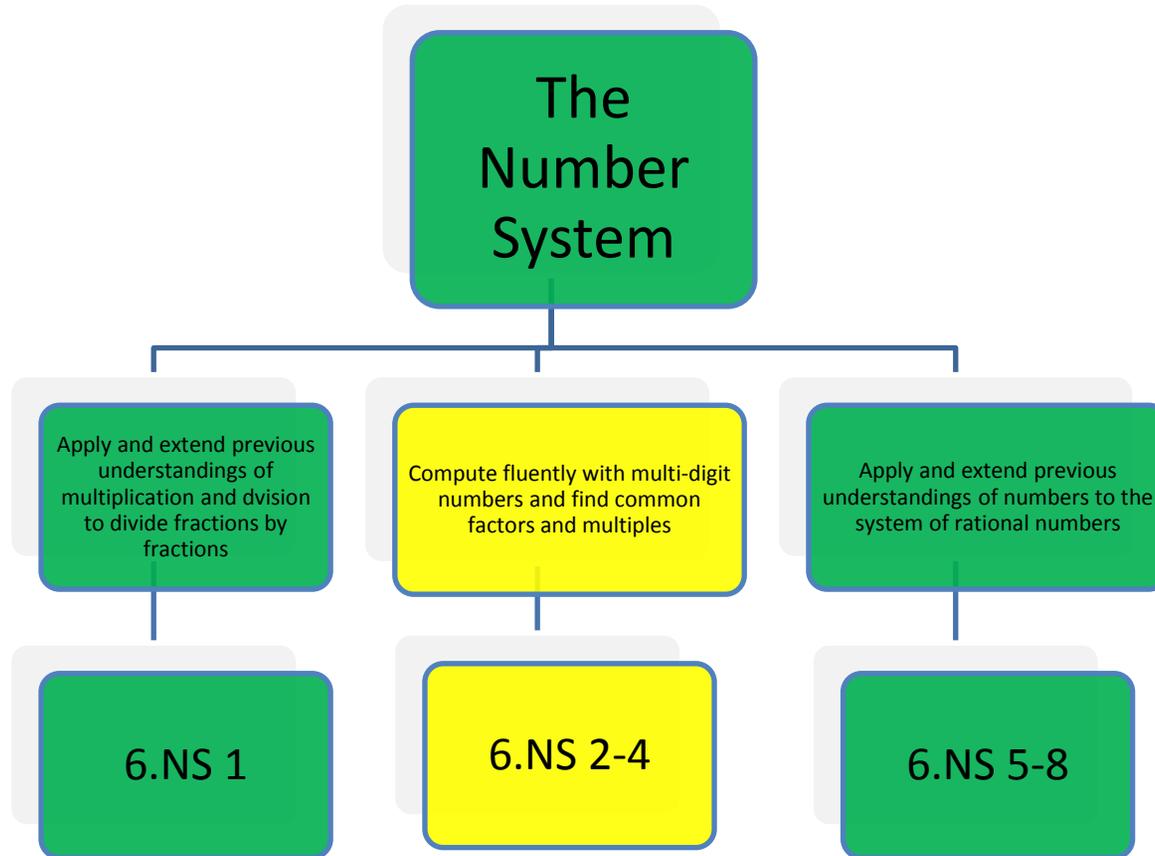


Grade 6  
Unit 2  
The Number System



**GRADE 6 – UNIT 2**  
**The Number System**

**Critical Area:** Description of the critical area: Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

CLUSTERS	COMMON CORE STATE STANDARDS
<p><b>(m)<sup>1</sup> Apply and extend previous understandings of multiplication and division to divide fractions by fractions.</b></p>	<p><b>Number System</b></p> <p><b>6.NS.1.</b> Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. <i>For example, create a story context for <math>(2/3) \div (3/4)</math> and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that <math>(2/3) \div (3/4) = 8/9</math> because <math>3/4</math> of <math>8/9</math> is <math>2/3</math>.</i> <i>(In general, <math>(a/b) \div (c/d) = ad/bc</math>.) How much chocolate will each person get if 3 people share <math>1/2</math> lb of chocolate equally? How many <math>3/4</math>-cup servings are in <math>2/3</math> of a cup of yogurt? How wide is a rectangular strip of land with length <math>3/4</math> mi and area <math>1/2</math> square mi?</i></p>
<p><b>(m)<sup>1</sup> Compute fluently with multi-digit numbers and find common factors and multiples.</b></p>	<p><b>6.NS.2.</b> Fluently divide multi-digit numbers using the standard algorithm.</p> <p><b>6.NS.3.</b> Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.</p> <p><b>6.NS.4.</b> Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. <i>For example, express <math>36 + 8</math> as <math>4(9 + 2)</math>.</i></p>
<p><b>(m)<sup>1</sup> Apply and extend previous understandings of numbers to the system of rational numbers.</b></p>	<p><b>6.NS.5.</b> Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.</p> <p><b>6.NS.6.</b> Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.</p> <p>a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., <math>-(-3) = 3</math>, and that 0 is its own opposite.</p>

	<p>b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.</p> <p>c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.</p> <p><b>6.NS.7.</b> Understand ordering and absolute value of rational numbers.</p> <p>a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. <i>For example, interpret <math>-3 &gt; -7</math> as a statement that <math>-3</math> is located to the right of <math>-7</math> on a number line oriented from left to right.</i></p> <p>b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. <i>For example, write <math>-3^{\circ}\text{C} &gt; -7^{\circ}\text{C}</math> to express the fact that <math>-3^{\circ}\text{C}</math> is warmer than <math>-7^{\circ}\text{C}</math>.</i></p> <p>c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. <i>For example, for an account balance of <math>-30</math> dollars, write <math> -30  = 30</math> to describe the size of the debt in dollars.</i></p> <p>d. Distinguish comparisons of absolute value from statements about order. <i>For example, recognize that an account balance less than <math>-30</math> dollars represents a debt greater than 30 dollars.</i></p> <p><b>6.NS.8.</b> Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.</p>
<p><math>(s/a)^2</math> Solve real-world and mathematical problems involving area, surface area, and volume.</p>	<p><b>Geometry</b></p> <p>6.G.1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.</p> <p>6.G.2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas <math>V = lwh</math> and <math>V = bh</math> to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.</p>
<p><b>MATHEMATICAL PRACTICES</b></p>	<p><b>PROGRESSION</b></p>
<ol style="list-style-type: none"> <li>1. <b>Make sense of problems and persevere in solving them.</b></li> <li>2. Reason abstractly and quantitatively.</li> <li>3. <b>Construct viable arguments and critique the arguments of others.</b></li> <li>4. <b>Model with mathematics.</b></li> <li>5. Use appropriate tools strategically.</li> </ol>	<p>6-7, Ratios and Proportional Relationships  <a href="http://commoncoretools.files.wordpress.com/2012/02/ccss_progression_rp_67_2011_11_12_corrected.pdf">http://commoncoretools.files.wordpress.com/2012/02/ccss_progression_rp_67_2011_11_12_corrected.pdf</a></p>

6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	
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<sup>1</sup> **Major Clusters – area of intensive focus where students need fluent understanding and application of the core concepts.**

<sup>2</sup> **Supporting/Additional Clusters – designed to support and strengthen areas of major emphasis/expose students to other subjects.**

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
<ul style="list-style-type: none"> <li>Procedures used for dividing fractions can be logically explained in several ways.</li> <li>The system of rational numbers includes negative numbers as well as positive ones.</li> <li>Rational number can be arranged in order.</li> <li>Absolute value can be described in more than way, depending upon the real-world context. It can be distance, or it can be size (magnitude).</li> <li>Points can be graphed in all four quadrants of a coordinate grid by using ordered pairs to determine location.</li> <li>A rational number can be represented as a point on a number line and the number line can be used as a tool to order rational numbers.</li> </ul>	<ol style="list-style-type: none"> <li>How can you compute fractions by using visual fraction models and equations?</li> <li>How do you find the GCF of two whole numbers using the distributive property?</li> <li>How do you use positive and negative numbers to describe quantities having opposite values?</li> <li>What is a rational number and how can you graph it?</li> <li>What is absolute value?</li> <li>How can you apply inverse operations in solving problems?</li> </ol>	<ul style="list-style-type: none"> <li>Absolute value</li> <li>Common factor</li> <li>Coordinate</li> <li>Distance</li> <li>Distributive property</li> <li>Divisor/Dividend</li> <li>Equivalent fractions</li> <li>Factors</li> <li>Fraction</li> <li>Greatest common factor</li> <li>Inequality</li> <li>Integers</li> <li>Least common multiple</li> <li>Magnitude</li> <li>Multiples</li> <li>Negative</li> <li>Number line</li> <li>Opposite</li> <li>Ordered pair</li> <li>Positive</li> <li>Quadrants</li> <li>Quotient</li> <li>Rational number</li> <li>Reflection</li> <li>Zero</li> </ul>

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
Materials: <b>Unit Planning Template</b> <a href="http://edtech4schools.pbworks.com/f/UbDPages.pdf">http://edtech4schools.pbworks.com/f/UbDPages.pdf</a>  <b>Supporting Teachers with Deep Understanding of Math Content</b> Ma, Liping. <i>Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United States</i> , Chapter 3 “Generating Representations: Division by Fractions”  <b>Other Resources</b> <a href="#">6.NS.1 Video explanation of division of fractions</a> <a href="#">Invert and Multiply?</a>	<ul style="list-style-type: none"> <li>Use of number line</li> <li>Use of human graph</li> <li>Using common denominators to divide fractions</li> <li>Journal / Quick Write Prompts</li> <li>Use of visual fraction models for division</li> <li>Using common denominators to divide fractions to understand the remainder</li> <li>Sorting cards</li> </ul>	<b>Formative Assessment</b> PARCC - Fraction Model <a href="http://www.parcconline.org/sites/parcc/files/FractionModelFINAL.pdf">http://www.parcconline.org/sites/parcc/files/FractionModelFINAL.pdf</a>  <b>LAUSD Assessments</b>  District assessments can be accessed through: <a href="http://achieve.lausd.net/math">http://achieve.lausd.net/math</a> <a href="http://achieve.lausd.net/ccss">http://achieve.lausd.net/ccss</a>

<p><b>Math Playground</b> <a href="#">6.NS.4 Factor Trees</a>  <b>Math Forum</b> <a href="#">6.NS.6 Graphing</a>  <b>Shodor</b> <a href="#">6.NS.6c Maze Game</a></p> <p><b>LAUSD Adopted Textbooks</b>  <b>LAUSD Adopted Textbooks</b></p> <ul style="list-style-type: none"> <li>• <a href="#">California Mathematics</a></li> <li>• <a href="#">College Preparatory Mathematics</a></li> <li>• <a href="#">Go Math</a></li> </ul> <p>Click on each list above for Textbook Alignment</p> <p><b>Illustrative Mathematics</b></p> <ul style="list-style-type: none"> <li>• <a href="#">6.NS Cup of Rice</a></li> <li>• <a href="#">6.NS Dan's Division Strategy</a></li> <li>• <a href="#">6.NS Interpreting a Division Computation</a></li> </ul> <p><b>NCTM Illuminations Lessons</b></p> <ul style="list-style-type: none"> <li>• 6.NS.6 Fractional Clothesline  <a href="http://illuminations.nctm.org/LessonDetail.aspx?id=L784">http://illuminations.nctm.org/LessonDetail.aspx?id=L784</a></li> <li>• 6.NS.4 The Product Game  <a href="http://illuminations.nctm.org/LessonDetail.aspx?id=U100">http://illuminations.nctm.org/LessonDetail.aspx?id=U100</a></li> <li>• The Venn Factor <a href="http://illuminations.nctm.org/LessonDetail.aspx?id=L859">http://illuminations.nctm.org/LessonDetail.aspx?id=L859</a></li> <li>• 6.NS.5 Zip, Zilch, Zero  <a href="http://illuminations.nctm.org/LessonDetail.aspx?id=L819">http://illuminations.nctm.org/LessonDetail.aspx?id=L819</a></li> </ul>	<ul style="list-style-type: none"> <li>• Fraction bars in teaching equivalent fractions</li> <li>• Vocabulary Development – 3x3 EL puzzle</li> </ul>	<p>Use your Single Sign On to access the Interim Assessments</p> <p><b>State Assessments</b></p> <p>California will be administering the SMARTER Balance Assessment as the end of course for grades 3-8 and 11. The 11th grade assessment will include items from Algebra 1, Geometry, and Algebra 2 standards. For examples, visit the SMARTER Balance Assessment at: SBAC - <a href="http://www.smarterbalanced.org/">http://www.smarterbalanced.org/</a></p>
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**LANGUAGE GOALS for low achieving, high achieving, students with disabilities and English Language Learners**

Students will be able to compare and contrast using a graphic organizer and orally multiplication and division of rational numbers.

*Example:* To express  $4 \times 5 = 20$  as division problem, I \_\_\_\_\_.

Students will be able to explain (writing/speaking) their understanding of absolute value and critique the reasoning of others.

*Example:* The absolute value of -5 is \_\_\_\_\_. This mean that if I travel to school for 5 miles, it will take \_\_\_\_\_ miles to travel home.

Students will be able to read a word problem and understand the situation in order to solve the problem.

Students will use the meaning of fractions to explain (writing/speaking) why the procedures for dividing fractions make sense.

*Example:* To divide fractions, I will \_\_\_\_\_ and \_\_\_\_\_.

When dividing fractions, students will be able to explain the meaning of the remainder.

*Example:* When I divide fraction, the remainder means \_\_\_\_\_.

**PERFORMANCE TASKS**

**Mathematics Assessment Project**

[6.NS.4 Pedro’s Tables](#)

[6.NS Interpreting Multiplication and Division](#)

[Adding and Subtracting Directed Numbers](#)

**Inside Mathematics**

[Winning Lines](#)

**LAUSD Concept Lessons**

[Fraction of a Fraction](#)

[Linking Fractions](#)

**Illustrative Mathematics**

[6.NS Jumping Flea](#)

[6.NS Above and below sea level](#)

[6.NS Integers on the Number Line 2](#)

[6.NS Fractions on the Number Line](#)

[6.NS Comparing Temperatures](#)

[6.NS Distances between Points](#)

**DIFFERENTIATION **

**UDL/ FRONT LOADING**

Students apply and extend their understanding of number sense, computation with multi-digit whole numbers and decimals (to hundredths), including application of order of operations, addition, subtraction, multiplication, and division of common fractions, and familiarity with factors and multiples.

Front load vocabulary associated with applications of integers such as:

- Thermometer
- Elevator
- Credit/Debit
- Sea level

**ACCELERATION**

- Have students describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge.
- Students design a story problems using temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge.
- Explain absolute value by using the distant they travel to school each way (to and fro). That distance is always positive.
- Provide a scenario where students will gather real – world data and graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

**INTERVENTION**

Intervention for low achieving students and students with disabilities:

- Small teacher to student ratio discussion
- Emphasize think-pair-share
- Make connections to real life
- Give concrete examples
- Use of manipulatives – especially the number line
- Use of multiple representations to represent fraction division problems. Set the problem in context and represent the problem with a concrete or pictorial model.
- Provide multiple experiences to understand the relationships between numbers, absolute value, and statements about order.

		<ul style="list-style-type: none"> <li>• Example: in real world, the absolute value can be used to describe size or magnitude. An ocean depth of 900 feet, write <math> -900  = 900</math> to describe the distance below sea level</li> </ul>
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**References:**

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2. McCallum, W., Zimba, J., Daro, P. (2011, December 26 Draft). *Progressions for the Common Core State Standards in Mathematics*. Cathy Kessel ( Ed.). Retrieved from <http://ime.math.arizona.edu/progressions/#committee>.
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7. California Department of Education. (2013). Draft Mathematics Framework Chapters. Retrieved from <http://www.cde.ca.gov/be/cc/cd/draftmathfwchapters.asp>.
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9. The University of Arizona. (2011-12). Progressions Documents for the Common Core Math Standards. Retrieved from <http://ime.math.arizona.edu/progressions>.