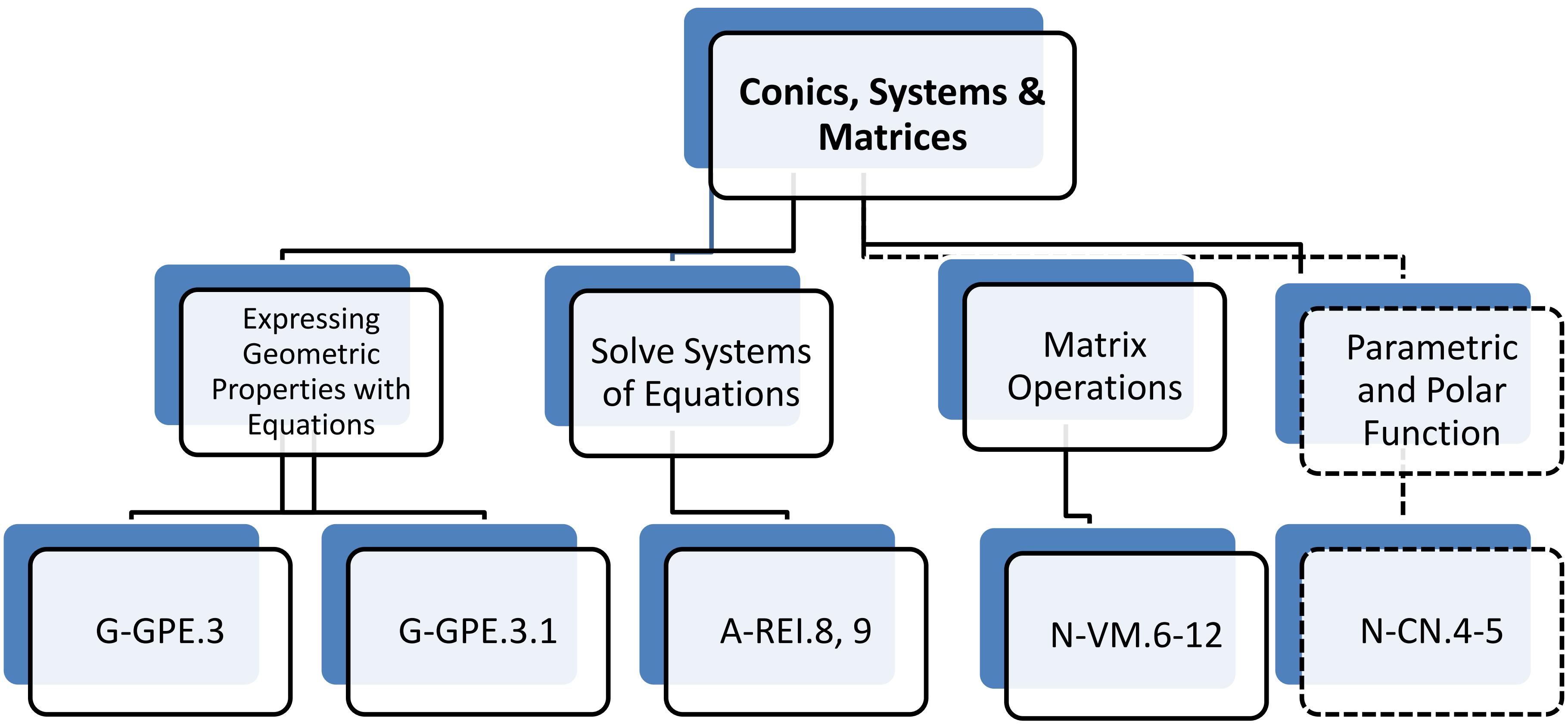
Precalculus
Unit 4
Matrices and Conic Section



Precalculus – UNIT 4
Conics, Systems and Matrices

Critical Area: Students derive the equations of ellipses and hyperbolas given foci. Given a quadratic equation of the form $ax^2 + by^2 + cx + dy + e = 0$, they use the method of completing the square to put the equation in standard form; identify whether the graph of the equation is a circle, parabola, ellipse, or hyperbola as well as graph the equation. Students model situations, involving payoffs in games, economic, or geometric situations to systems of linear equations and connect the newfound knowledge of matrices to solving problems. Students investigate vectors as geometric objects in the plane that can be represented by ordered pairs, and matrices as objects that act on vectors. Students discover that vector addition and subtraction behave according to certain properties, while matrices and matrix operations observe their own set of rules. Students discover with matrices a new set of mathematical objects and operations among them that has a multiplication that is not commutative. Students expand the skills involved in working with equations into several areas: trigonometric functions, by setting up and solving equations such as sin 2ϴ = ½; parametric functions by making sense of the equations $x = 2t$, $y = 3t + 1$, $0 \le t \le 10$.

CLUSTERS	COMMON CORE STATE STANDARDS
Translate between the geometric and the equation for a conic section	**Geometry – Expressing Geometry Properties with Equations** **G-GPE.3**. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is consistent. **G-GPE.3.1**. Given a quadratic equation of the form $ax^2 + by^2 + cx + dy + e = 0$, use the method of completing the square to put the equation in standard form; identify whether the graph of the equation is a circle, parabola, ellipse, or hyperbola, and graph the equation.
Solve systems of equations	**Algebra – Reasoning with Equations and Inequalities** **A-REI.8**. (+) Represent a system of linear equations as a single matrix equation in a vector variable. **A-REI.9**. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equation (using technology for matrices of dimension 3 x 3 or greater).
Represent complex numbers and their operations on the complex plane	**Number and Quantity – Complex Number** **N-CN.4.** (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. **N-CN. 5.** (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\,i)^3 = 8$ *because* $(-1 + \sqrt{3}\,i)$ *has modulus* 2 *and argument* 120°.

Perform operations on matrices and use matrices in applications	**Number and Quantity – Vector and Matrix Quantities** **N-VM.6**. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. **N-VM.7**. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. **A-VM.8**. Add, subtract, and multiply matrices of appropriate dimensions. A-VM.9. Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. **A-VM.10**. Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix and multiplicative inverse. **A-VM.12**. Work with 2 x 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.
MATHEMATICAL PRACTICES	PROGRESSION
1. Make sense of problems and persevere in solving them. **2. Reason abstractly and quantitatively.** **3. Construct viable arguments and critique the arguments of others.** **4. Model with mathematics.** 5. Use appropriate tools strategically. 6. Attend to precision. **7. Look for and make use of structure.** 8. Look for and express regularity in repeated reasoning.	

★Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.

(+) Indicates additional mathematics to prepare students for advanced courses.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
• The sum or difference of the distances of the foci from the directrix is consistent. • Graphs of quadratic equations of the form $ax^2 + by^2 + cx + dy + e = 0$ can be circles, parabolas, ellipses, or hyperbolas • The inverse of a matrix may or may not exist. • Matrices could be used to solve real-world problems involving system of linear equations. • Linear equations can be represented as a single matrix • The equations of ellipses and hyperbolas can be derived from the foci.	1) What are the geometric characteristics of conics? 2) How do you identify the graphs of quadratic equations of the form $ax^2 + by^2 + cx + dy + e = 0$? 3) How do you find the inverse of a matrix? 4) How can data be represented as a matrix? 5) How can the equations of ellipses and hyperbolas derived from the foci? 6) How would you use matrices to solve system of equations?	• circle • Cramer's Rule • determinant • directrix • eccentricity • ellipses • foci • hyperbolas • identity matrix • inverse matrix • Law of Sine/Cosine • matrix • parabola • parametric function • row/column • scalar • vector

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
NCTM Illuminations • *Cutting Conics:* G-GPE.3 Students explore and discover conic sections by cutting a cone with a plane. Circles, ellipses, parabolas, and hyperbolas are examined using the Conic Section Explorer tool. Physical manipulatives such as dough can optionally be used as well. http://illuminations.nctm.org/Lesson.aspx?id=2907 • *Human Conics:* G-GPE.3 Students use sidewalk chalk and rope to illustrate the locus definitions of ellipses and parabolas. Kinesthetics, teamwork, and problem solving are stressed as students take on the role of focus, directrix, and point on the conic, and figure out how to construct the shape. http://illuminations.nctm.org/Lesson.aspx?id=3003 • *Mars Orbit:* F-IF.10	Students will explore the conic sections and describe how to cut a cone to create the various conic sections. Separate the class into 6 groups (or a multiple of 6 if your class is large). Assign two conic sections to each group. There are 6 different ways to do this: circle/ellipse, circle/hyperbola, circle/parabola, ellipse/hyperbola, ellipse/parabola, and hyperbola/parabola. Each group should create a poster summarizing what they've learned about their two conic sections and comparing and contrasting them. Students will write a summary of either the ellipse or parabola construction for the benefit of a classmate who has missed the lesson. The summary should include the definition and an explanation of how the drawing technique applies the definition. Afterwards, students can exchange and critique their summary with other students. Given parametric equations, group students and ask them to find the polar equation that will give the same shape as the	1. Ask students to describe how they discovered how to cut their cones to create each conic section – circles, ellipses, parábolas, hyperbolas. 2. Give students a picture of an ellipse and a parabola with possible foci or directrix indicated. Ask them to use a ruler and right angle measure to determine and explain whether or not the figure is actually the named conic. 3. Using data regarding the distance from the Sun and the orbital periods of other planets, ask students to generate parametric equations for the orbits of the other planets in the solar system relative to the Earth.

Students will generate parametric equations to describe the position of planets relative to the Sun; then, they will combine the equations to describe the position of Mars relative to Earth.
http://illuminations.nctm.org/Lesson.aspx?id=3980

- *Caesar Cipher*: A-VM.10
  In this lesson, students will investigate the Caesar substitution cipher. Text will be encoded and decoded using inverse operations.
  http://illuminations.nctm.org/Lesson.aspx?id=1926

- *Pick's Theorem as a System of Equations*: N-VM.6
  Students will gather three examples from a Geoboard or other representation to generate a system of equations. The solution will provide the coefficients for Pick's Theorem.
  http://illuminations.nctm.org/Lesson.aspx?id=2089

LAUSD Adopted Textbooks

Precalculus Enhanced with Graphing Utilities, 4th Edition , Sullivan & Sullivan, Pearson/Prentice Hall (2005).

Precalculus Graphical, Numerical, Algebraic, 7th edition, Demana, Waits, Foley & Kennedy, Addison Wesley, Pearson Education (2007).

Pre-Calculus with Limits: A Graphing Approach, 5th edition, Larson, Hostetler, and Edwards, Houghton/Mifflin, Boston/New York (2008).

Precalculus with Trigonometry Concepts and Applications, 2nd edition, Foerster, Key Curriculum (2007)

one obtained with given parametric equations. Afterwards, students will share their explanations in a whole class discussion.

Guide students to transform a system of linear equations in two variables into matrix. Then help students to solve the resulting matrix by Cramer's rule. For example:

$2x+3y=6$
$4x+5y=1$
$\left|\begin{matrix}2 & 3 & \vert 6\\ 4 & 5 & \vert 1\end{matrix}\right|$ and then solved using determinants.

Design an instruction that would help students to discover with matrices a new set of mathematical objects and operations among them that has a multiplication that is not commutative.

Engage students in an activity to investigate vectors as geometric objects in the plane that can be represented by ordered pairs, and matrices as objects that act on vectors.

Have them discover that vector addition and subtraction behave according to certain properties, while matrices and matrix operations observe their own set of rules.

LANGUAGE GOALS

Writing:

1) Students will explain and justify the process of completing the square to identify whether the quadratic equation of the form $ax^2 + by^2 + cx + dy + e = 0$ is a ellipse, circle, parabola, or a hyperbola.
   Example: I completed the process of completing the square by _____and found that _____. This means that graph of the quadratic equation is a _______.

2) Students will compare and contrast the differences and similarities between ellipses, circles, parabolas, and hyperbolas.
   Example: If the eccentricity of a conic section is _______, than the graph is a _______.

Listening and Speaking:

1) Students will generate class discussions using specific vocabulary related to solving systems of equations and operations with matrices.
   Example: To solve systems of equations by (matrix), I can use the (Cramer rule) and l transform the equation in matrix and then find the determinant.
2) Students will explain and justify how to solve systems of equations using operations with matrices to a partner as well as restating and summarizing their partner's explanation.
   Example: First I __________ because ____________, second I ___________ because__________,…

Reading:

1) Students will identify the relevant information and details in a passage that help them to use matrices to represent and manipulate data.

PERFORMANCE TASKS

Textbook: Larson, R. and Hostetler, R. (2007). Pre-Calculus with Limits, 5th edition. Boston, New York: Houghton/Mifflin.

Publisher: Houghton Mifflin Company
Authors: Larson, R., Hostetler, R.

Topic: Matrices and Systems of Equations
- Healthcare (Page 601, Problem 70)
- Data Analysis: License Drivers (Page 610, Problem 72)
- Data Analysis: Supreme Court (Page 630, Problem 58)

Topic: Vectors
- Navigation (Page 459, Problem 84)
- Braking Load (Page 468, Problem 67)

Topic: Conics
- Suspension Bridge (Page 742, Problem 62)
- Loran (Page 761, Problem 42)
- Satellite Tracking (Page 798, Problem 58)
-

Textbook: Pre-Calculus with Limits: A Graphing Approach, 5th edition, Larson, Hostetler, and Edwards, Houghton/Mifflin, Boston/New York, 2008.

- Earthquake: Page 667 #35

- Road Design: Page 669 #94
- Architecture: Page 678 #49
- Navigation: Page 688 #46
- Planetary Motion: Page 727 #55

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<tr><th>FRONT LOADING</th><th>ACCELERATION</th><th>INTERVENTION</th></tr>
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• Introduce students to ellipses and help them understand that conics are like circle and parabolas.

• Introduce students to the equations and graphs of conics and help them see the relationship between equation and graph.

• Engage students in an activity that would connect their understanding of conics to the real-world.

• Design a frontloading activity that would introduce students to the idea of vector addition and subtraction and matrix operations.
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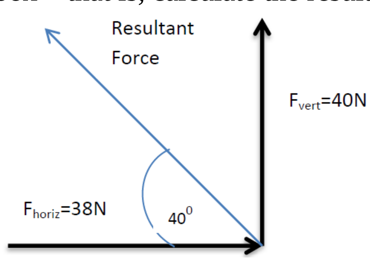
• Provide examples of real-world problems that can be modeled by circles, parabolas, and ellipses.

• Students will write and graph equations in polar form.

• Students will classify conics from their general equation.

• Students will use properties of parabolas, ellipses, and hyperbolas to model and solve real-life problems.

• Through guided discovery, have students discover that vector addition and subtraction behave according to certain properties, while matrices and matrix operations observe their own set of rules.

• Have students resolve vectors involving forces. For example: Two students are moving a box up a ramp that is inclined 40^0. One pushes on the box with a force of 40 N. The other student pulls the box with a force of 38 N at an angle of 40° from horizontal. What is the net force (magnitude and direction) on the box – that is, calculate the resultant force.


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• Have students use calculators or computer software to lessen the computational burden in simplifying and graphing conics.

• Use hands-on activities to allow students to explore how conics may vary (i.e. Using a string and two thumbtacks, have students explore how to obtain ellipses that are long or narrow)

• Have students use technology to perform matrix operations; including addition, subtraction, and multiplication of matrices.
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References:

1. National Governors Association Center for Best Practices, Council of Chief State School Officers. (2010). *Common Core State Standards (Mathematics)*. Washington D.C.: National Governors Association Center for Best Practices, Council of Chief State School Officers.
2. McCallum, W., Zimba, J., Daro, P. (2011, December 26 Draft). *Progressions for the Common Core State Standards in Mathematics*. Cathy Kessel (Ed.). Retrieved from http://ime.math.arizona.edu/progressions/#committee.
3. Engage NY. (2012). New York Common Core Mathematics Curriculum. Retrieved from http://www.engageny.org/resource/high-school-algebra-i.
4. Mathematics Assessment Resource Service, University of Nottingham. (2007 - 2012). Mathematics Assessment Project. Retrieved from http://map.mathshell.org/materials/index.php.
5. Smarter Balanced Assessment Consortium. (2012). Smarter Balanced Assessments. Retrieved from http://www.smarterbalanced.org/.
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7. California Department of Education. (2013). Draft Mathematics Framework Chapters. Retrieved from http://www.cde.ca.gov/be/cc/cd/draftmathfwchapters.asp.
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10. Larson, R. and Hostetler, R. (2007). *Pre-Calculus with Limits*, 5th edition. Boston, New York: Houghton/Mifflin.
11. Sullivan, M. & Sullivan III, M. (2006). *Precalculus Enhanced with Graphing Utilities*, 4th edition. New Jersey: Pearson, Prentice Hall.
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13. Foerster, P. A. (2007). *Precalculus with Trigonometry Concepts and Applications*, 2nd edition. Emeryville, CA: Key Curriculum.