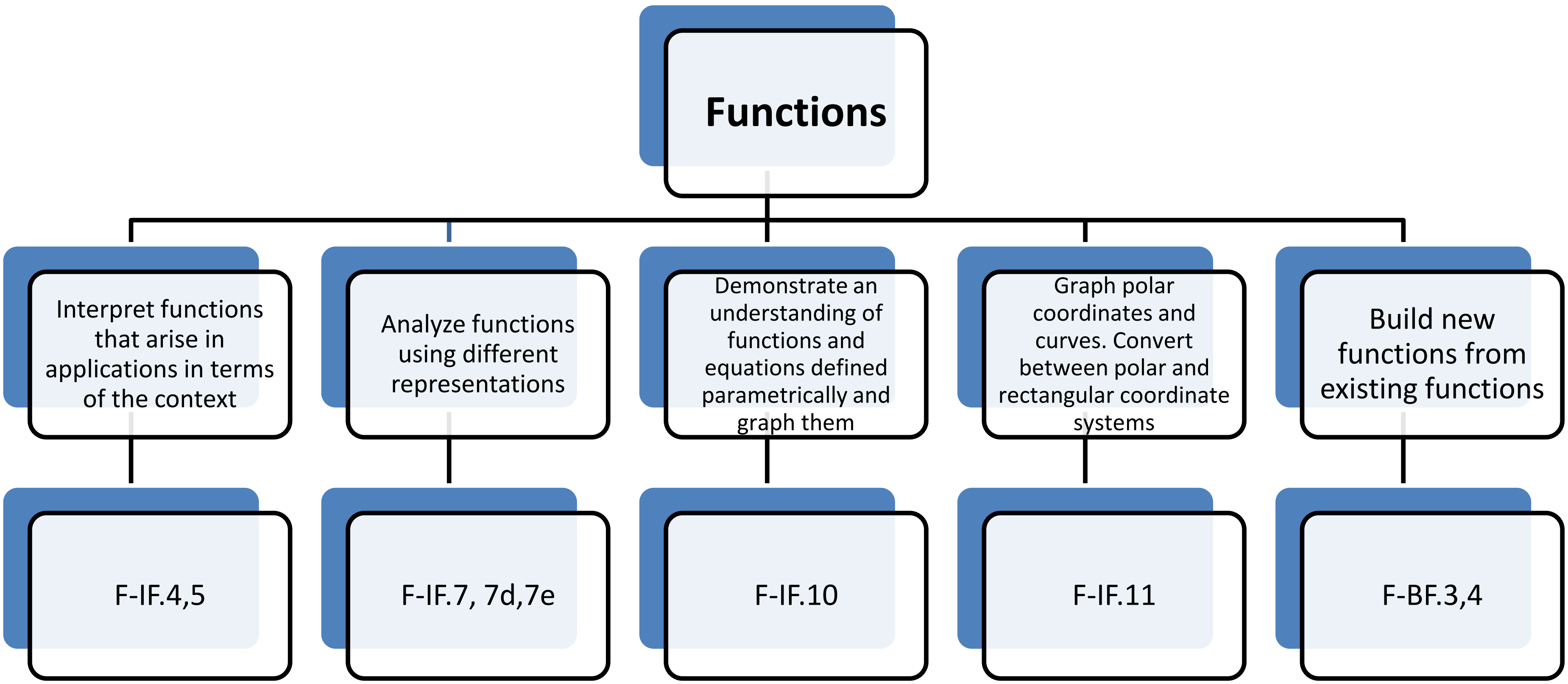
Precalculus
Unit 2
Functions



Precalculus – UNIT 2
Functions

Critical Area: Students develop conceptual knowledge of functions that set the stage for the learning of other standards in Precalculus. Students apply the standards in Interpreting Functions and Building Functions in the cases of polynomial functions of degree greater than two, more complicated rational functions, the reciprocal trigonometric functions, and inverse trigonometric functions. Students will examine end behavior of functions and learn how to find asymptotes. Students further their understanding of inverse functions and construct inverse functions by appropriately restricting domains. They also investigate the relationship between the graphs of sine and cosine as a function of theta and also use the parametric form of the functions where $x(\theta) = \cos(\theta)$ and $y(\theta) = \sin(\theta)$.

CLUSTERS	COMMON CORE STATE STANDARDS
Interpret functions that arise in applications in terms of the context	**Functions – Interpreting Functions** **F-IF.4.** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★ **F-IF.5.** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.★ **F-IF.7d.** (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. **F-IF.7e**. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. **F-IF.10. (+)** Demonstrate an understanding of functions and equations defined parametrically and graph them. CA **F-IF.11. (+)** Graph polar coordinates and curves. Convert between polar and rectangular coordinate systems. CA
Building a function that models a relationship between two quantities.	**Functions – Building Functions** **F-BF.3.** Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

CLUSTERS	COMMON CORE STATE STANDARDS
	F-BF.4. Find inverse functions. **b.** (+) Verify by composition that one function is the inverse of another. **c.** (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. **d.** (+) Produce an invertible function from a non-invertible function by restricting the domain.
MATHEMATICAL PRACTICES	**PROGRESSION**
1. **Make sense of problems and persevere in solving them.** 2. **Reason abstractly and quantitatively.** 3. Construct viable arguments and critique the arguments of others. 4. **Model with mathematics.** 5. **Use appropriate tools strategically.** 6. **Attend to precision.** 7. **Look for and make use of structure.** 8. Look for and express regularity in repeated reasoning.	http://opi.mt.gov/PDF/CCSSO/MCCS-MATH/STAGE1/Resources/2012_12-04Draft-High-School-Progression-Functions.pdf http://commoncoretools.me/wp-content/uploads/2013/07/ccss_progression_modeling_2013_07_04.pdf

★**Indicates a modeling standard linking mathematics to everyday life, work, and decision-making.**

(+) Indicates additional mathematics to prepare students for advanced courses.

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
• Different types of relationships between quantities can be modeled with different types of functions. • Functions and relations can be represented using polar coordinates. • Functions and equations can be defined parametrically. • All functions have algebraic, numerical, graphical and verbal representations. • Operations and transformations apply to all types of functions and can be used to build new functions from existing functions. • Graphs are visual representations of solution sets of equations and inequalities. • The inverse functions interchange the domain and the range.	1. What relationships exist between quantities that can be modeled by functions? 2. How can functions and relations be represented using polar coordinates? 3. Why is it important to define functions and equations parametrically? 4. What does it mean to solve equations graphically? 5. What do the domain and the range of a function represent? 6. What do the maximum and minimum represent and how do they relate to the end behavior of a function? 7. What do asymptotes represent? 8. How do we build new functions from existing functions using transformations? 9. What are the similarities and differences	• asymptotes - horizontal, vertical, oblique • composite function • compress/ stretch • data • domain • end behavior • exponential • functions • increasing/decreasing • intercepts • inverse function • invertible, non-invertible • logarithmic

ENDURING UNDERSTANDINGS	ESSENTIAL QUESTIONS	KEY VOCABULARY
• The domain of a non-invertible function needs to be restricted in order to construct its inverse function. • Graphs of functions can explain the observed local and global behavior of a function. • Asymptotes represent the restricted domain or range. • The graph of a function demonstrates the end behavior as it approaches the vertical, horizontal or oblique asymptotes. • Real world situations can be modeled and solved by using various functions.	between linear, quadratic, exponential, logarithmic and polynomial functions? 10. How do we compare/contrast exponential and logarithmic functions? 11. What are inverse functions and what are they being used for? 12. How do we restrict the domain of a non-invertible function to produce an invertible function?	• logistic • maximum/minimum • modeling • one-to-one functions • periodicity • polynomial • range • rational • reflection over the x and y-axis • relationship • restricted domain • shift • symmetry • transformations • vertical/ horizontal

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
Illustrative Mathematics • The Canoe Trip, Variation 2 - F-IF.4 http://www.illustrativemathematics.org/illustrations/394 • Transforming the graph of a function - F-BF.3 http://www.illustrativemathematics.org/illustrations/742 • Building an Explicit Quadratic Function by Composition - F-BF.3 www.illustrativemathematics.org/illustrations/744 • Graphic Representations of the Real Life Situations http://graphingstories.com/ • Relating the Domain of a Function to its Graph- Asymptotes and Restricted Domains http://learnzillion.com/lessonsets/456-relate-the-domain-of-a-function-to-its-graph-accounting-for-asymptotes-and-	The domains for Unit 2 are Interpreting Functions and Building Functions. Students are required to understand families of functions and the inverse of those functions. Students must be familiar with the concept and formal definition of inverse functions, namely that if $f \circ g(x) = g \circ f(x) = x$, then $f(x)$ and $g(x)$ are inverses of one another. Teachers should first work with evaluating functions, then composing general functions and finally composing inverse functions. Once students have mastered the composition of inverse functions, they should be made to derive the inverse functions and prove that they have found the inverse by using the above definition. Students should recall parent functions $f(x)$ and then explore the effect of $f(x)+k$, $f(x+k)$, $kf(x)$, $f(kx)$ on the graph for all k. The mathematical	SBAC – http://www.smarterbalanced.org/ PARCC - http://www.parcconline.org/sites/parcc/files/HighSchoolAlg2Math3-GraphsofFunctions.pdf http://www.parcconline.org/sites/parcc/files/BRHSSampleItem.pdf

RESOURCES	INSTRUCTIONAL STRATEGIES	ASSESSMENT
restricted-domains **LAUSD Adopted Textbooks** Precalculus Enhanced with Graphing Utilities, 4th Edition , Sullivan & Sullivan, Pearson/Prentice Hall (2005). Precalculus Graphical, Numerical, Algebraic, 7th edition, Demana, Waits, Foley & Kennedy, Addison Wesley, Pearson Education (2007). Pre-Calculus with Limits: A Graphing Approach, 5th edition, Larson, Hostetler, and Edwards, Houghton/Mifflin, Boston/New York (2008). Precalculus with Trigonometry Concepts and Applications, 2nd edition, Foerster, Key Curriculum (2007)	progressions demand that students are fluent with the parent functions and can use them quickly to determine the graph of transformed functions. Students will explore the relationship between functions and their inverses on the same coordinate plane. They will use that understanding to then explain the connection between the line of symmetry of the two functions and the algebraic method of letting $f(x) = x$ and $x = f^{-1}(x)$ to solve for the inverse function $f^{-1}(x)$. Students should then come to understand why a function needs to be one-to-one in order to have an inverse and then why it is necessary and possible to restrict a domain on a function to create an invertible function. Provide visual examples of transformed functions while manipulating different constants in the function parameters. Have students use technology to manipulate the parameters of the functions and record how the parameters affect the graphs and tables of the functions. An instructional conversation with all students, in particular English learners will benefit from scaffolds that promote use of academic language. Mathematically Speaking is a scaffold that may be used. http://camsp.net/documents/NCTM-SpeakingArticle.pdf	

LANGUAGE GOALS
Writing: 1) Students will explain and justify in writing the behavior of the function as it approaches horizontal and vertical asymptotes. *Example: As the function approaches positive infinity along the x-axis, the graph of the function approaches the horizontal asymptote from above.* 2) Students will explain (in writing and orally) the effects of transformations on a function and test that understanding for all parent functions. *Example: The transformation* $f(x+a)+b$, *moves the parent function* $-a$ *units in the horizontal direction and* b *units in the vertical direction.*

LANGUAGE GOALS
3) Students will compare and contrast (in writing and orally) the differences and similarities between linear, polynomial, and exponential functions. *Example: All three functions increase as x increases. Polynomial and exponential functions are curves and the linear function is a line. Exponential functions will increase at a faster rate than polynomial functions.* 4) Students will write about the relationship between the inverse of functions and the concept of rotating the axes about the line of symmetry to determine the

inverse function.

Example: The inverse function can be determined by rotating the function of the graph about the line of symmetry. This is algebraically equivalent to interchanging the x and y values in a function and solving for y.

5) Students will write about how functions can be used in real life to facilitate repeated algorithms.

Example: Computers often make use of functions to run programs i.e. clicking on the icon for Internet Explorer will run a function to launch a program that connects the modem to the internet and opens a screen to a preselected page.

Listening and Speaking:

1) Students will participate in class discussions using specific vocabulary related to transformations and functions.
2) Students will explain and justify (orally) how to graph a function to a partner as well as restating and summarizing their partner's explanation.

Example: First I ___________ because _____________, second I ____________ because___________,…

Reading:

1) Students will identify the relevant information and details in a passage and create a single function that represents a composition out of many subparts.

PERFORMANCE TASKS

Precalculus Enhanced with Graphing Utilities, Sullivan & Sullivan, 4th Edition (2005), **ISBN-10:** 0131490923

F-IF.4.

- Enclosing the Most Area with a Fence, Page 166, 79
- Minimizing Marginal Cost, Page 144, # 87
- Norman Windows, Page 167, # 86

F-IF.7d

- Population Model, Page 197, # 54
- Cost of a Can, Page 210, # 61

F-BF.4

- Discussion and Writing, Page 270, # 84-90

Precalculus Graphical, Numerical, Algebraic, 7th edition, Demana, Waits, Foley & Kennedy, Addison Wesley, Pearson Education 2007

F-IF.4.

- Modeling the Height of a Bouncing Ball, Chapter 2 Project, page 273
- Designing a Swimming Pool, Page 255, # 38

F-IF.7d

- Designing a Cardboard Box, Page 265, # 59

PERFORMANCE TASKS

- Industrial Design, Page 272, # 94 and 95
- Designing a Juice Can, Page 265, # 61

Illustrative Mathematics

Transforming the graph of a function - F-BF.3: http://www.illustrativemathematics.org/illustrations/742

Building an Explicit Quadratic Function by Composition - F-BF.3: www.illustrativemathematics.org/illustrations/744		
DIFFERENTIATION		
FRONT LOADING	ACCELERATION	INTERVENTION
• Have students recall how to graph by hand linear, quadratic and cubic functions from a table of values and then understand how to graph all parent functions. • Involve students in a simple discussion of what an inverse means and how differs from opposite or reciprocal. • Involve students in the processes required to solve equations and start to discuss the concept of inverse functions. • Engage students in an activity that would involve comparing linear functions with quadratics functions, and then quadratics functions and exponential functions. • Have students match linear, quadratic, and exponential functions with their graphs, tables, and equations. • Get the students to explain how to solve quadratic equations by completing the square.	• Students work in small groups with a curriculum that is conceptually demanding as well as rigorous due to the speed at which the course moves and the concepts covered. Students collaborate and concentrate on tasks for extended periods of time, to contribute to discussions, to predict and test their predictions. • The assessments for advanced students will demand the ability to apply learned concepts to solving abstract or real world problems or summarize the patterns/ concepts learned. Students will use the "Socratic Method" for posing questions to discover connections, patterns and structure. • Students learn about the modeling of real world data with polynomial functions, rational functions, exponential functions, radical functions, logarithmic functions, and sinusoidal functions. They explore in depth the various characteristics of functions, i.e. rates of change, concavity, inverses, continuity, discontinuity and asymptotes. Students further explore functions in terms of composite and inverse functions, their transformations and periodicity. • Students work on projects to apply these concepts to real-world problems by creating equations and exploring the graphs of those equations using technology application to determine which parts of the graph are relevant to the problem context. A modeling problem involving the most efficient way to solve an investment dilemma for stock broker will be posed to the students. They model the problem by finding a solution pathway that would optimize the profit of the brokerage firm when they invest in different stock options. Given the stock pricing which would involve volume-based discount, students will model and graph the revenue, cost, and profit functions. They would interpret the vertices, intercepts, and intersection points as well as solve	• Demonstrate for students how to create tables of values and to use those values to generate the graph of the function. • Allow students to use technology to quickly generate a table of values after they have shown some skill in evaluating expressions by hand. • Students use graphic organizers to graph functions by hand and analyze the graphs in terms of increasing, decreasing, positive, or negative; relative maximums/ minimums, the symmetry and continuity. • Using technology, students work in small groups to graph different function and compare/contrast the graphs and make conclusions. • Students graph transformations of quadratic and cubic functions graphs and analyze the differences and similarities.

	systems of equations to discover the exact number of customers that would help them maximize revenue and profit.	

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2. Smarter Balanced Assessment Consortium. (2012). Smarter Balanced Assessments. Retrieved from http://www.smarterbalanced.org/
3. National Council of Teachers of Mathematics (NCTM) Illuminations. (2013). Retrieved from http://illuminations.nctm.org/Weblinks.aspx
4. Partnership for Assessment of Readiness for College and Career. (2012). PARCC Assessments. Retrieved from http://www.parcconline.org/parcc-assessment.
5. Engage NY. (2012). New York Common Core Mathematics Curriculum. Retrieved from http://www.engageny.org/resource/high-school-pre-calculus
6. The University of Arizona. (2011-12). Progressions Documents for the Common Core Math Standards. Retrieved from http://ime.math.arizona.edu/progressions
7. Graduate NYC! Curriculum Alignment Project. Retrieved from http://gradnyc.com/wp-content/uploads/2013/04/FINAL-Math-HS-Functions-Unit-v2.pdf
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9. Demana, F.D., Waits, B.K., Foley, G.D., & Kennedy, D. (2007). *Precalculus Graphical, Numerical, Algebraic*, 7th edition. Addison Wesley, Pearson Education.
10. Larson, R.; Hostetler, R.; and Edwards, B. H. (2008). *Pre-Calculus with Limits: A Graphing Approach*, 5th edition. Boston, New York: Houghton/Mifflin.
11. Foerster, P. A.(2007). *Precalculus with Trigonometry Concepts and Applications*, 2nd edition. Emeryville, CA: Key Curriculum.