

In this document we provide two different ways of adapting the language of the practice standards to the K–5 setting.

In this section we provide annotated versions of the standards that provide additional interpretation of the standards appropriate for K–5 classrooms. This section is intended for people who want to understand how the original language of the standards applies in K–5.

In the next section we provide *elaborations* of the standards: narrative descriptions that integrate the annotations from the first section and provide a coherent description of how the practice standards play out in the K–5 classroom.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

- For example, younger students might notice that when tossing two-color counters to find combinations of a given number, they always get what they call "opposites"—when tossing 6 counters, they get 2 red, 4 yellow and 4 red, 2 yellow and when tossing 4 counters, they get 1 red, 3 yellow and 3 red, 1 yellow.

- Students in the middle elementary grades might notice a pattern in the change to the product when a factor is increased by 1: $5 \times 7 = 35$ and $5 \times 8 = 40$ —the product changes by 5; $9 \times 4 = 36$ and $10 \times 4 = 40$ —the product changes by 4. Students might then express this regularity by saying something like, "When you change one factor by 1, the product increases by the other factor."

- Mathematically proficient elementary students formulate conjectures about what they notice, for example, that when 1 is added to a factor, the product increases by the other factor; or that, whenever they toss counters, for each combination that comes up, its "opposite" can also come up. As students practice articulating their observations, they learn to communicate with greater precision (MP.6). As they explain why these generalizations must be true, they construct, critique, and compare arguments (MP.3).

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y .

- Mathematically proficient students at the elementary grades use structures such as place value, the properties of operations, other generalizations about the behavior of the operations (for example, the less you subtract, the greater the difference), and attributes of shapes to solve problems. In many cases, they have identified and described these structures through repeated reasoning (MP.8). For example, when younger students recognize that adding 1 results in the next counting number, they are identifying the basic structure of whole numbers. When older elementary students calculate 16×9 , they might apply the structure of place value and the distributive property to find the product: $16 \times 9 = (10 + 6) \times 9 = (10 \times 9) + (6 \times 9)$. To determine the volume of a $3 \times 4 \times 5$ rectangular prism, students might see the structure of the prism as five layers of 3×4 arrays of cubes.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students at the elementary grades look for regularities as they solve multiple related problems, then identify and describe these regularities. For example, students might notice a pattern in the change to the product when a factor is increased by 1: $5 \times 7 = 35$ and $5 \times 8 = 40$ —the product changes by 5; $9 \times 4 = 36$ and $10 \times 4 = 40$ —the product changes by 4. Students might then express this regularity by saying something like, “When you change one factor by 1, the product increases by the other factor.” Younger students might notice that when tossing two-color counters to find combinations of a given number, they always get what they call “opposites”—when tossing 6 counters, they get 2 red, 4 yellow and 4 red, 2 yellow and when tossing 4 counters, they get 1 red, 3 yellow and 3 red, 1 yellow. Mathematically proficient students formulate conjectures about what they notice, for example, that when 1 is added to a factor, the product increases by the other factor; or that, whenever they toss counters, for each combination that comes up, its “opposite” can also come up. As students practice articulating their observations, they learn to communicate with greater precision (MP.6). As they explain why these generalizations must be true, they construct, critique, and compare arguments (MP.3).

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