

CCSS-Aligned Mathematical Task

Title: Angel's Multiplication Error, Grade 5

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Task

This is Angel's work on a multiplication problem:

$$\begin{array}{r} \overset{4}{3} \overset{5}{7} \\ \overset{3}{7} \overset{3}{1} \\ \mathbf{179} \\ \mathbf{\times 64} \\ \mathbf{716} \\ + \mathbf{1,074} \\ \hline \mathbf{1,790} \end{array}$$

1. Use estimation to explain why Angel's answer is not reasonable.
2. Find 179×64 using a correct version of Angel's method (standard algorithm). Then show another way of solving the problem to help Angel see why your answer is correct. Use any strategy to show your solution.
3. What error do you think Angel made? What concept(s) does Angel not understand and still need to work on?

Rationale for Lesson

This task is designed to help students explore a very common multiplication error that occurs when using the standard algorithm. Students often forget a step or misalign the addends (partial products) if you aren't thinking about the value of a particular digit. Place value understanding is the primary driver of this lesson. The magnitude of the factors must be maintained in the algorithm. Students practice analyzing the reasonableness of an answer, as well as develop flexibility in solving multi-digit multiplication problems, which is an aspect of fluency.

Common Core State Standards for Content

5.NBT.1 Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $\frac{1}{10}$ of what it represents in the place to its left.

5. NBT.5 Fluently multiply multi-digit whole numbers using the standard algorithm.

Common Core State Standards for Mathematical Practice

MP1 Make sense of problems and persevere in solving them.

MP2 Reason abstractly and quantitatively.

MP3 Construct viable arguments and critique the reasoning of others.

MP6 Attend to precision .

5th grade students may demonstrate these Math Practices in the following ways:

- Explain calculations based upon models, properties of operations and rules that generate patterns. (MP2, MP3)
- Construct arguments using visual models, such as objects and drawings. (MP3)
- Refine their mathematical communication skills as they participate in mathematical discussions involving questions like “How did you get that?” and “Why is that true?” (MP3)
- Explain their thinking to others and respond to others’ thinking. (MP3)
- Use various strategies to solve problems and they defend and justify their work with others. (MP1, MP2, MP3)

DOK Level:

Level 2 Skill/Concept

Level 3 Strategic Thinking/Reasoning

Enduring Understandings

Students will:

- Understand that the value of a particular digit is based on its place in the number, and in fact, that is one of the advantages of the standard algorithm.
- Understand the importance of being able to analyze mistakes and construct reasonable, evidence-based arguments to justify their thinking.

Materials Needed

Student Handout

Poster paper for charting or copy of divided Circle Map (without answers) to project.

Differentiation/Support:

Several access strategies to support different types and levels of learners have

been incorporated into the lesson, including:

- collaborative group work
- discussion strategies (Rally Robin in the Set Up phase, Think-Pair-Share, Stronger and Clearer Each Time in problem 3)
- Manipulatives: base ten blocks, place value chart/mat, graph paper, place value disks
- Multiple strategies: area model, distributive property, partial products

Other ways to differentiate might include the following:

- Change the numbers in the problem to simpler numbers
- Vary the expectations for writing outcome
- Sentence frames for oral/written responses
- A possible extension for early finishers might be to generate their own incorrect problem and solution and have a partner analyze their error.
- Small groups could be formed to target specific content skills or to strengthen mathematical practices. Small groups can employ a variety of scaffolding including teacher guided instruction, peer-coaching, or use of manipulatives.

DAY 1

Set-Up Phase (10-15 min.)

Begin by conducting a class discussion about “estimation” and “reasonableness.” First, use the Rally Robin protocol (or another of your choice) for brainstorming (see below). Divide the class into two groups.

Assign one of the following questions to each half of the room:

- 1) What do you know about estimation?
- 2) What do you know about reasonableness?

Students will brainstorm for their question in small groups of 3 or 4. At the end, call on students to share, writing their responses on the divided circle map. If they haven't surfaced, make sure to establish accurate understandings of the two terms. In addition, the situations that require estimation might be discussed to add context to the definitions.

Rally Robin

Purpose: In groups, students stand up and face each other and alternate generating brief oral responses.

Procedure:

1. Students partner up in groups of 3 or 4.
2. Teacher poses the question or topic.
3. Teacher sets the timer for a designated amount of time.

- Each student takes turns responding to the question or topic until time is called. This is intended to be a brainstorming session, not a discussion.

In order to engage your students and create “buy-in,” establish the purpose of the lesson, which is to help another student correct a mathematical mistake. Have a short discussion around why this might be useful, and why it is beneficial for mathematicians to be able to find, analyze and correct an error.

Give students Problem 1. Read and discuss the problem. The following is a sample of how to use “Talk Moves” to check for understanding of the task:

Have students silently read the problem. Then have a student read the problem aloud, or read it chorally. Have the students’ pair-share what the problem is asking them to do. Ask a volunteer to restate what the problem is asking. Then ask a volunteer to clarify or add-on to what the previous student stated. Ask if there are any remaining questions before moving on. Repeat the procedure of checking for understanding of the problem for question 2.

Explore Phase (20-30 min.)

For each of the 2 questions, allow the students five minutes of alone think time before they begin working with a partner(s). Let go and allow students time and any manipulatives needed.

Possible Student Strategies	Focusing Questions	Assessing Questions	Advancing Questions
Algorithm	What is the problem asking you to do?	How did you get your answer?	How is estimation connected to the correct answer?
Partial Product Area Model	What do you know already?	How do you know you are correct?	In what other situations could you use your error correcting strategy?
Partial Product Distributive Property	Why is Angel’s answer unreasonable?		Where else have we used this strategy?
Place Value Chart	How can this tool help you solve the problem?		

Share, Discuss, and Analyze Phase

As the students work in their groups, the teacher chooses 3 or 4 groups to share their solution paths with the whole class. The order of sharing can depend on your specific emphasis and grouping of kids. Generally the teacher is the facilitator of the Share/Discuss/Analyze step and would guide the sharing and the discussions of the students. You may want to show a progression of complexity of student responses during this time. For example, for problem 1 show two different examples where students showed estimation to check Angel's error. For problem 2 try to find students that used partial products and area model to solve the problem another way. These suggestions were generated based on student responses in our field test of this lesson.

Include an Academic Language Word Wall, adding to the list of posted words as the students use them in their discussions, providing support to all students.

Include the 'Talk Moves' strategies to increase discourse. Ask students to restate and add-on what other students are saying. This will also help keep everyone engaged. Encourage the students to ask questions during this step to deepen their understanding of the solution path presented.

Day 2

Set-Up Phase (5-10 min.)

Remind students of their work on Angel's Error done previously. Briefly review problems 1 and 2 to remind students of the strategies they used. Review the Academic Language Word Wall from the previous day's work.

Give students Problem 3. Read and discuss the problem. The following is a sample of how to use "Talk Moves" to check for understanding of the task:

Have students silently read the problem. Then have a student read the problem aloud, or read it chorally. Have the students' pair-share what the problem is asking them to do. Ask a volunteer to restate what the problem is asking. Then ask a volunteer to clarify or add-on to what the previous student stated. Ask if there are any remaining questions.

Explore Phase (30-40 min.)

Introduce the 'Stronger and Clearer Each Time' activity from Jeff Zwiers' book called, Common Core Standards in Diverse Classrooms. The purpose

Conversation Skills is “that in peer conversations, they are able to take more ownership of ideas and learning.”

Allow 10-15 minutes for students to talk to 3 different people and add answers to their paper.

Stronger & Clearer Each Time (Please refer to student handout)

- 1) First have students write their initial response to the given question.
- 2) Have them talk to at least 3 people. Each time they talk to a partner, they **take notes** on what he/she says.
- 3) Each time they talk to a partner, they can build from and borrow the ideas and language of previous partners. Encourage students to make their answer *stronger each time* with better evidence, examples, and explanations; and encourage them to try to make their idea *clearer each time* by using a topic sentence, logical ways to organize and link sentences, and precise words.
- 4) At the end, allow the students quiet think time to write their final responses to question 3 on the student handout (Angel’s Error). Make sure students understand that they can use ideas, evidence, or language they heard from their partners.

Share, Discuss, and Analyze Phase (15-20 min.)

As the students work on their responses to question 3, the teacher chooses 3 or 4 students to share their ideas of what error Angel made and what concepts he still needs to develop with the whole class. The order of sharing can depend on how well students can explain their reasoning. Generally the teacher is the facilitator of the Share/Discuss/Analyze step and would guide the sharing and the discussions of the students. You may want to show a progression of complexity of student responses during this time.

Application

You are calculating 107×56 , and the calculator shows this: 952
Is that right? How do you know? Explain to your partner how estimation can help to check your work.

Summary

In this lesson students explored finding and correcting the errors in the work of others. Students should also come away from this lesson with a better understanding of how estimation can be used as a tool to determine the reasonableness of an answer to help them check their own work. In addition, there are multiple ways to solve any problem and being able to explain your problem solving strategy is key to becoming a proficient problem solver.

Quick-Write

What did you learn in this lesson about correcting mistakes one can make using

Here is the answer using the partial products algorithm:

$$\begin{array}{r} 179 \\ \times 64 \\ \hline \end{array}$$
$$\begin{array}{r} 9 \times 4 = 36 \\ 70 \times 4 = 280 \\ 100 \times 4 = 400 \\ 9 \times 60 = 540 \\ 70 \times 60 = 4200 \\ 100 \times 60 = \underline{+6000} \\ 11,456 \end{array}$$

Here is a rectangle with side lengths $100+70+9$ and $60+4$ that shows all of the partial products as the area of part of the rectangle:

	100	70	9
60	$60 \times 100 =$ 6,000	$60 \times 70 =$ 4,200	$60 \times 9 =$ 540
4	$4 \times 100 =$ 400	$4 \times 70 =$ 280	$4 \times 9 =$ 36

$$\begin{array}{r}
 6,000 \\
 4,200 \\
 540 \\
 400 \\
 280 \\
 + \quad 36 \\
 \hline
 11,456
 \end{array}$$

Clearer and Stronger Each Time

Name _____

1. **Topic:** What error do you think Angel made? What concept(s) does Angel not understand and still need to work on?

My initial response: _____ _____ _____ _____

Stronger & Clearer Each Time

- 5) Talk to at least 3 people. Each time you talk to a partner, take notes on what he/she says.
- 6) Each time you talk to a partner, you can build from and borrow the ideas and language of previous partners. Try to make your answer *stronger each time* with better evidence, examples, and explanations; and try to make your idea *clearer each time* by using a topic sentence, logical ways to organize and link sentences, and precise words.
- 7) At the end, on the back of the paper write a final response to the topic. You can use ideas, evidence, or language you heard from your partners.

Question/Prompt:	What error do you think Angel made? What concept(s) does Angel not understand and still need to work on?
Partner #1's name: _____	<i>Ideas, evidence, and language from this person that will help my idea to be stronger and clearer:</i>
Partner #2's name: _____	<i>Ideas, evidence, and language from this person that will help my idea to be stronger and clearer:</i>
Partner #3's name: _____	<i>Ideas, evidence, and language from this person that will help my idea to be stronger and clearer:</i>

Name: _____ Date: _____

This is Angel's work on a multiplication problem:

$$\begin{array}{r} 4 5 \\ 3 3 \\ 179 \\ \times 64 \\ \hline 716 \\ + 1,074 \\ \hline 1,790 \end{array}$$



1. Use estimation to explain why Angel's answer is not reasonable.

2. Find 179×64 using a correct version of Angel's method (standard algorithm). Then show another way of solving the problem to help Angel see why your answer is correct. Use any strategy to show your solution.

